On the construction of corrected Light-Front Hamiltonian for QED_2

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Abstract

The counterterms, which must be included into Light-Front Hamiltonian of QED_2 to get the equivalence with conventional Lorentz-covariant formulation, are found. This is done to all orders of perturbation theory in fermion mass, using the bosonization at intermediate steps and comparing Light-Front and Lorentz-covariant perturbation theories for bosonized model. The obtained Light-Front Hamiltonian contains all terms, present in the QED_2 theory, canonically (naively) quantized on the Light-Front (in the Light-Front gauge) and an unusual counterterm. This counterterm is proportional to linear combination of fermion zero modes (which are multiplied by some operator factors neutralizing their charge and fermionic number). The coefficients before these zero mode operators are UV finite and depend on condensate parameter in the θ-vacuum. These coefficients are proportional to fermion mass, when this mass goes to zero.

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1. Introduction

Hamiltonian approach to Quantum Field Theory in Light-Front (LF) coordinates [1] $x^{\pm} = (x^0 \pm x^3)/\sqrt{2}$, $x^{\perp} = (x^1, x^2)$, with x^+ playing the role of time, is one of nonperturbative approaches which can be used in attempts to solve strong coupling problems [2–4]. One quantizes field theory on the hyperplane $x^+ = 0$ and considers the generator P_+ of the shift along the x^+ axis as a Hamiltonian H. The generator of the shift along the axis x^-, i. e. the momentum operator P_-, does not displace the surface $x^+ = 0$, where the quantization is performed, and is kinematic (according to Dirac terminology) in contrast to dynamical generator P_+. Operator P_- does not depend on the interaction and is quadratic in field variables. Due to spectral condition it is nonnegative and has zero eigenvalue on the physical vacuum. Positive and negative p_- parts of Fourier modes of fields play a role of creation and annihilation operators on this vacuum and can be used to form LF Fock space. Therefore the physical vacuum can be trivially described in terms of this "mathematical" Fock space vacuum. The spectrum of bound states can be found by solving the Schroedinger equation

$$P_+|\Psi\rangle = p_+|\Psi\rangle \tag{1}$$

in the subspaces with fixed $p_-, p_{\perp}$. The mass m can be found as $m^2 = 2p_+p_- - p_{\perp}^2$. The procedure of solving this bound state problem can be nonperturbative. This was demonstrated for (1+1)-dimensional field theory models by so called DLCQ method [5–7] (see [3]).

LF Hamiltonian formalism faces with specific divergences at $p_- = 0$ [2–4, 8], and needs a regularization. Simplest translationally invariant regularization is the cutoff $p_- \geq \varepsilon > 0$. This cutoff breaks Lorentz and gauge symmetries. Another regularization, retaining gauge invariance, is the cutoff $|x^-| \leq L$ with periodic boundary conditions for fields. In this case the momentum p_- is discrete, $p_- = p_n = (\pi n/L)$, $n = 0, 1, 2, ...$, and zero modes, $p_- = 0$, are well separated. Canonical formalism allows in principle to express these zero modes in terms of other, nonzero modes via solving constraints (usually it is a very complicated nonlinear problem) [9–13].

Examples of nonperturbative calculations in (1+1)-dimensional models, using canonical LF formulation, show that the description of vacuum effects can be nonequivalent to that in the conventional formulation in Lorentz coordinates [14–16]. Furthermore lowest order perturbative calculations show

a difference between corresponding LF and Lorentz-covariant Feynman diagrams [17, 18]. This can invalidate the conventional renormalization procedure for LF perturbation theory. These violations of the equivalence between Lorentz-covariant and canonical LF formulations can be caused by the breakdown of Lorentz and other symmetries due to LF regularizations described above. Nevertheless for nongauge theories, like Yukawa model, it is possible to restore the equivalence, at least perturbatively, by adding few simple counterterms to canonical LF Hamiltonian [18–21]. For gauge theories we have used the general method of the paper [21], to compare the Feynman diagrams of LF and conventional (covariant) perturbation theories (to all orders) and to construct via this method counterterms for canonical LF Hamiltonian. It appears that the number of such counterterms under simple $|p_-| \geq \varepsilon$ regularization is infinite [21]. We are able to overcome this difficulty only via a complication of the regularization scheme (using Pauli-Villars type "ghost" fields) [22]. The theory on the LF with the resulting QCD-Hamiltonian of [22] is perturbatively equivalent in the limit of removing the regularization to the Lorentz-covariant QCD. But this LF Hamiltonian is rather complicated and contains several unknown parameters. Furthermore it does not guarantee that the description of nonperturbative vacuum effects will be correct.

One can explicitly analyse these vacuum effects only in two-dimensional gauge models. Inspite of the simplicity of 2-dimensional formulation one may hope to get some indications how to describe the nonperturbative vacuum effects also in four dimensions.

In (1+1)-dimensional space-time it is possible to go beyond usual perturbation theory by transforming gauge field model, like QED_2, to scalar field model ("massive Sine-Gordon" type model for the QED_2) [23–26]. This transformation includes the "bosonization" procedure, i. e. the transition from fermionic to bosonic variables [16, 27–30]. After such transformation of the Hamiltonian the fermionic mass term plays the role of the interaction term in corresponding scalar field theory. Therefore the perturbation theory in terms of this scalar field (with fermion mass becoming the coupling constant) plays the role of conventional chiral perturbation theory. Furthermore nontrivial description of quantum vacuum in QED_2, related with instantons and the "θ" vacua [25, 26, 31, 32], is taken into account in this scalar field theory simply by the appearing of the "θ"–parameter in the interaction term.

In the present paper we use this formulation to construct correct LF QED_2 Hamiltonian, applying perturbation theory (to all orders in coupling, i.e. in

fermion mass) in corresponding scalar field theory. After obtaining corrected boson form of LF Hamiltonian we make the transformation to original canonical LF form (with fermionic field variables on the LF) plus some new terms ("counterterms").

Due to nonpolynomial character of boson field interaction, in perturbation theory one faces ultraviolet (UV) divergencies for some infinite sums of diagrams of a given order [33]. That is why it is difficult to apply straightforwardly the method of the paper [21] of the comparison of LF and covariant perturbation theories. In the paper [33] we used Pauli-Villars regularization for scalar field. This allowed to apply the general method of [21] and to find the counterterms that correct the boson LF Hamiltonian [33] (analogous consideration for sine-Gordon model not containing UV-divergences was carried out in the paper [34]). These counterterms are proportional to chiral condensate parameter, which depends on the UV cutoff and becomes infinite after removing the regularization (while perturbatively one can see how this infinity cancels the divergencies appearing in LF perturbation theory). The LF Hamiltonian corrected in this way depends explicitly on the UV cutoff parameter. Furthermore we can not return simply to original canonical LF variables (the transformation from boson to fermion variables on the LF becomes complicated due to the presence of boson Pauli-Villars ghost fields in the chosen regularization scheme).

In the present paper we avoid these difficulties using, instead of boson Pauli-Villars UV regularization, simple UV cutoff at intermediate steps. This becomes possible, because we are able to prove UV finiteness of Lorentz-covariant Green functions of the considered scalar field theory (without vacuum loops) to all orders of perturbation theory in fermion mass parameter. We carry out a detailed comparison of LF and Lorentz-covariant perturbation theories to all orders. As a result, we find counterterms to canonical LF Hamiltonian restoring its equivalence with Lorentz-covariant formulation (in terms of boson field). The form of the obtained LF Hamiltonian allows simple transformation to original canonical fermion variables on the LF and the direct comparison with the original canonical LF Hamiltonian.

The resulting form of LF Hamiltonian includes a term, which appears under naive canonical LF quantization of QED_2 in LF gauge and a new counterterm proportional to linear combination of fermion zero modes (multiplied by some operator factors neutralizing their charge and fermionic number) with the UV finite coefficients, depending on condensate parameter. These coefficients are

4

proportional to fermion mass, when this mass approaches to zero.

One can use this LF Hamiltonian for nonperturbative calculations applying DLCQ method and fitting condensate parameters to known spectrum.

However it is more interesting to extract from this study some features that could be true also in the 4-dimensional QCD. This was attempted recently in [35, 36].

In Sect. 2 we review Hamiltonian formulation of QED_2 and the canonical form of the bozonization transformation. In Sect. 3 we prove the UV finiteness of the bosonized Lorentz-covariant perturbation theory to all orders in fermion mass. In Sect. 4-6 we compare LF and Lorentz-covariant perturbation theories in boson form and construct counterterms generating the difference between them. At the beginning of the Sect. 4 a more detailed explication of the content of the Sect. 4-6 is given. In Sect. 7 we rewrite the corrected boson LF Hamiltonian in terms of canonical fermion variables on the LF.

2. Hamiltonian formulation of QED_2 in Lorentz frame and the bosonization transformation

Let us start from usual Lagrangian density for QED_2 in Lorentz coordinates $x^\mu = (x^0, x^1)$:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^m D_\mu - M)\Psi, \tag{2}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $D_\mu = \partial_\mu - ieA_\mu$, $A_\mu(x)$ are gauge fields, $\Psi = \binom{\psi_-}{\psi_+}$, $\bar{\Psi} = \Psi^+\gamma^0$ are fermion fields with mass M, e is the coupling constant, γ^μ are chosen as follows:

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad \gamma^5 = i\gamma^0\gamma^1 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \tag{3}$$

From this Lagrangian one gets the following Hamiltonian H:

$$H = \int dx^1 \left(\frac{1}{2}\Pi_1^2 + \sum_{r=\pm} r\left(\psi_r^+ iD_1\psi_r \right) + iM\left(\psi_+^+\psi_- - \psi_-^+\psi_+ \right) \right), \tag{4}$$

where $\Pi_1 = \frac{\partial L}{\partial(\partial_0 A_1)} = F_{01}$ is the momentum conjugate to A_1. Besides one gets the constraint equation

$$\partial_1\Pi_1 + e\left(\psi_+^+\psi_+ + \psi_-^+\psi_- \right) = 0. \tag{5}$$

It is convenient to consider the theory on the interval $-L \leq x^1 \leq L$ with periodic boundary conditions for fields and to fix the appropriate gauge [10–13]:

$$\partial_1 A_1 = 0. \tag{6}$$

Then all fields can be represented by corresponding Fourier series, and zero, $p_1 = 0$, mode can be separated from nonzero modes. We denote the zero and nonzero mode parts of a function $f(x^1)$ as follows:

$$f(x^1) \equiv f_{(0)} + [f(x^1)], \quad f_{(0)} \equiv \int \frac{dx^1}{2L} f(x^1). \tag{7}$$

Owing to the constraint (5) we get

$$[\Pi_1] = -e\partial_1^{-1}[\psi_+^+\psi_+ + \psi_-^+\psi_-], \tag{8}$$

$$Q \equiv \int\limits_{-L}^{L} dx^1 \left(\psi_+^+\psi_+ + \psi_-^+\psi_-\right) = 0. \tag{9}$$

We use eq-n (8) to exclude $[\Pi_1]$ from the Hamiltonian (4). The zero modes $A_1 \equiv A_{1(0)}$, $\Pi_{1(0)}$ remain independent and commute at fixed time as follows:

$$[A_{1(0)}, \Pi_{1(0)}] = \frac{i}{2L}. \tag{10}$$

Here and in the following we assume that $(\partial_1^{-1}[f(x^1)])_{(0)} \equiv 0$. We impose the condition (9) on physical states after quantization:

$$Q|phys\rangle = 0. \tag{11}$$

Quantum definition of the charge Q and current operators $I_\pm(x) = \psi_\pm^+(x)\psi_\pm(x)$ includes gauge invariant UV regularization [16, 28] (the "point splitting" with the parameter δ) of the following form:

$$I_r(x; \delta) = \psi_r^+(x^1 - \frac{ri\delta}{2})\psi_r(x^1 + \frac{ri\delta}{2})\exp(reA_1\delta), \quad r = \pm. \tag{12}$$

Here and in the following we set $x^0 = 0$ and omit this variable for brevity.

$$Q(\delta) = \int\limits_{-L}^{L} dx^1 \sum_{r=\pm} I_r(x; \delta). \tag{13}$$

This regularization makes the Hamiltonian well defined [28]. Indeed, the term of the Hamiltonian (4),

$$
H_{\bar{\Psi}D\Psi}(\delta) = \int\limits_{-L}^{L} dx^1 \sum_{r=\pm} r \left(\psi_r^+(x^1 - \frac{ri\delta}{2}) i D_1 \psi_r(x^1 + \frac{ri\delta}{2}) \exp(reA_1\delta) \right) =
$$
$$
= \frac{d}{d\delta}Q(\delta), \tag{14}
$$

can be minimized by filling negative "energy" levels (according to Dirac procedure). Following the work [28] one can find the minimizing state, starting from the state vector $|0_D\rangle$ such that

$$
\psi_r(x)|0_D\rangle = 0, \tag{15}
$$

using Fourier decomposition

$$
\psi_r(x) = \frac{1}{\sqrt{2L}} \sum_{n=-\infty}^{\infty} \psi_{n,r} e^{-ip_n x^1}, \quad p_n = \frac{\pi}{L}n \tag{16}
$$

and introducing a set of states $|q\rangle = |q_+, q_-\rangle$ with "energy" levels filled up to $q_\pm$:

$$
|q\rangle = \prod_{n=-\infty}^{q_+} \prod_{m=q_-}^{\infty} \psi_{n,+}^+ \psi_{m,-}^+ |0_D\rangle, \tag{17}
$$

where q_+, q_- are integers and

$$
[\psi_{n,r}^+, \psi_{m,r'}]_+ = \delta_{nm}\delta_{rr'}. \tag{18}
$$

One gets [16, 28]

$$
\int\limits_{-L}^{L} dx^1 I_r(x, \delta)|q\rangle = \left(\frac{L}{\pi\delta} + \left(rq_r + \frac{1}{2} + \frac{rLeA_1}{\pi} \right) \right) |q\rangle + O(\delta), \tag{19}
$$

$$
H_{\bar{\Psi}D\Psi}(\delta)|q\rangle =
$$
$$
= \left(-\frac{2L}{\pi\delta^2} - \frac{\pi}{12L} + \frac{\pi}{2L} \sum_{r=\pm} \left(rq_r + \frac{1}{2} + \frac{rLeA_1}{\pi} \right)^2 \right) |q\rangle + O(\delta). \tag{20}
$$

These expressions show that one can define the renormalized (i. e. UV finite in the $\delta \to 0$ limit) quantities as follows:

$$
I_r(x) = \lim_{\delta \to 0} \left(I_r(x; \delta) - \frac{1}{2\pi\delta} \right), \tag{21}
$$

$$Q = \lim_{\delta \to 0} \left(Q(\delta) - \frac{2L}{\pi \delta} \right) = \int_{-L}^{L} dx^1 \sum_{r=\pm} I_r(x), \tag{22}$$

$$H_{\bar{\Psi}D\Psi} = \lim_{\delta \to 0} \left(H_{\bar{\Psi}D\Psi}(\delta) + \frac{2L}{\pi \delta^2} + \frac{\pi}{12L} \right). \tag{23}$$

Moreover one can define the operators $Q_\pm$:

$$Q_r \equiv \left(\int_{-L}^{L} dx^1 I_r(x) \right) - \frac{1}{2} - \frac{rLeA_1}{\pi}, \qquad Q_+ + Q_- = Q - 1, \tag{24}$$

$$Q_r|q\rangle = rq_r|q\rangle. \tag{25}$$

According to the equation (19) (or (25)) and the paper [28] these operators are integer valued and commute with gauge fields and current operators:

$$[Q_r, \Pi_{1(0)}] = [Q_r, A_1] = [Q_r, I_{r'}(x)] = 0 \tag{26}$$

(let us remark that applying current operators to states $|q\rangle$, we can get the whole Hilbert space [37]). This definition leads to the commutation relation between Π_1 and $Q_5 = \int_{-L}^{L} dx^1 (I_+ - I_-)$:

$$[Q_5, \Pi_{1(0)}] = \frac{ie}{\pi} \tag{27}$$

because of eq-n (10). Renormalized currents (21) satisfy commutation relations with anomalous Schwinger term [14–16, 28]:

$$[I_r(x), I_{r'}(x')]_{x^0=0} = \frac{ir}{2\pi} \delta_{rr'} \partial_1 \delta(x^1 - x'^1). \tag{28}$$

These relations account for the axial vector anomaly in QED_2. On the other hand they are a base for the bosonization, because their form resembles boson field canonical commutation relations. Indeed one can introduce the following canonically conjugate boson field variables:

$$\Phi(x) = -\frac{\sqrt{\pi}}{e} \left(\Pi_{1(0)} - e\partial_1^{-1}[I_+ + I_-]_x \right) = -\frac{\sqrt{\pi}}{e} \Pi_1(x), \tag{29}$$

$$\Pi(x) = \sqrt{\pi} \left(I_+(x) - I_-(x) \right), \tag{30}$$

8

$$[\Phi(x), \Phi(x')]_{x^0=0} = [\Pi(x), \Pi(x')]_{x^0=0} = 0,$$
$$[\Phi(x), \Pi(x')]_{x^0=0} = i\delta(x^1 - x'^1). \tag{31}$$

Let us construct inverse transformation formula, that expresses initial fermion variables in terms of boson ones. With this aim we introduce [14–16, 28] operators $\omega_\pm$, canonically conjugated to the $Q_\pm$ (and commuting with other canonical variables):

$$[\omega_r, Q_{r'}] = i\delta_{rr'}. \tag{32}$$

According to the equality

$$e^{-i\omega_r} Q_r e^{i\omega_r} = Q_r + 1, \tag{33}$$

these operators can be represented by their action on the $|q\rangle$ states:

$$e^{i\omega_+}|q_+, q_-\rangle = |q_+ + 1, q_-\rangle, \qquad e^{i\omega_-}|q_+, q_-\rangle = |q_+, q_- - 1\rangle. \tag{34}$$

The expression for fermion fields can be written in the same way as it is done in string theory [16, 30]:

$$\psi_r(x) = N_0 e^{-i\omega_r} e^{ri\frac{\pi}{2}Q - ri\pi\frac{x^1}{L}Q_r} : e^{-i\sqrt{\pi}[\partial_1^{-1}[\Pi] + r\Phi]_x} :, \tag{35}$$

where N_0 is a normalization factor depending on the choice of normal ordering in (35). To fix this normal ordering we take Fourier decompositions of boson fields in the interaction picture form (i. e. in the free field like form with the mass $m = e/\sqrt{\pi}$ and operators a_n^+, a_n playing the role of creation and annihilation operators):

$$[\Phi(x)] = \frac{1}{\sqrt{4L}} \sum_{n\neq0} \frac{a_n e^{-ip_n x^1} + h.c.}{\sqrt{E_n}}, \tag{36}$$

$$[\Pi(x)] = \frac{1}{i\sqrt{4L}} \sum_{n\neq0} \sqrt{E_n} \left(a_n e^{-ip_n x^1} - h.c.\right), \tag{37}$$

$$[a_m, a_n^+] = \delta_{mn}, \qquad E_n \equiv \sqrt{m^2 + p_n^2}. \tag{38}$$

Then we take normal ordering with respect to operators a_n, a_n^+, and the factor N_0 becomes equal to

$$N_0 = \frac{1}{\sqrt{2L}} \exp\left(-\frac{\pi}{4L} \sum_{n>0} \frac{(E_n - p_n)^2}{p_n^2 E_n}\right). \tag{39}$$

9

One can check [16] (taking into account UV regularization (12) of the product of fermion fields) that operators (35) satisfy correct canonical anticommutation relations and reproduce the definitions (29),(30).

Having the explicit formulas for the bosonization one can rewrite the Hamiltonian (4) (where the substitution of (8) for the $[\Pi_1]$ is done) in terms of boson variables. One gets on the physical subspace of states which satisfy constraint (11), the following expression [14–16]:

$$H = \int\limits_{-L}^{L} dx^1 : \left(\frac{1}{2}\Pi^2 + \frac{1}{2}(\partial_1\Phi)^2 + \frac{m^2}{2}\Phi^2 - MN_1\cos(\omega + \sqrt{4\pi}\Phi) \right) : +$$

$$+const, \tag{40}$$

where $\omega \equiv \omega_+ - \omega_- - \sqrt{4\pi}\Phi_{(0)} - \pi/2$, $N_1 = \frac{1}{L}\exp\left(\frac{\pi}{L}\sum_{n>0}(\frac{1}{p_n} - \frac{1}{E_n})\right)$. This Hamiltonian coincides with the Hamiltonian of scalar field theory, where the field $\Phi(x)$ is periodic on the interval $-L \leq x^1 \leq L$ and the operator ω, commuting with all boson field variables, can be replaced by it's eigenvalue, playing the role of the "θ" – parameter. The relation between ω and the θ-parameter can be explained by noticing that the remaining (after fixing the gauge $\partial_1 A_1 = 0$) discrete gauge group

$$\Psi_n \xrightarrow[\Omega_l]{} \Psi_{n+l}, \qquad A_1 \xrightarrow[\Omega_l]{} A_1 + \frac{\pi l}{eL} \tag{41}$$

is realized by the operators $\Omega_l = \exp(il\omega)$, and that these gauge operators are responsible for the change of topological charge and connected with the definition of the θ-vacuum [26, 32]. The operators a_n, determined by eq-ns (36),(37), annihilate free boson field vacuum $|0\rangle$:

$$a_n|0\rangle = 0. \tag{42}$$

This vacuum state can be described also in terms of states (17) [16, 28].

In the limit $mL \to \infty$ one gets [16]

$$\lim_{mL\to\infty} N_1 = \frac{me^C}{2\pi}, \tag{43}$$

where $C = 0.577216\ldots$ is the Euler constant, and the Hamiltonian (40) takes the form corresponding to Lorentz-invariant theory of scalar field Φ:

$$H = \int dx^1 : \left(\frac{1}{2}\Pi^2 + \frac{1}{2}(\partial_1\Phi)^2 + \frac{m^2}{2}\Phi^2 - \frac{Mme^C}{2\pi}\cos(\theta + \sqrt{4\pi}\Phi) \right) : . \tag{44}$$

This scalar field theory is considered in the sections 3-6.

3. The proof of the UV finiteness of the Lorentz-covariant boson theory to all orders in fermion mass

Let us start with Lagrangian corresponding to the Hamiltonian (44):

$$L = L_0 + L_I, \tag{45}$$

$$L_0 = \frac{1}{8\pi} \left(\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2 \right), \tag{46}$$

$$L_I = \frac{\gamma}{2} e^{i\theta} : e^{i\varphi} : + \frac{\gamma}{2} e^{-i\theta} : e^{-i\varphi} :, \quad \gamma = \frac{Mme^C}{2\pi}, \tag{47}$$

where for convenience we use the notation $\varphi = \sqrt{4\pi}\Phi$; the normal ordering symbol means that in the perturbation theory in γ one excludes all diagrams with lines starting and ending at the same vertex (it also corresponds to usual normal ordering of the operators in a interaction Hamiltonian). Let us consider the structure of Feynman diagrams in Lorentz coordinates for this theory. There are two types of vertices with j lines, the 1st type giving the factor $i^{j+1} e^{i\theta} \gamma/2$ and the 2nd giving the factor $i^{-j+1} e^{-i\theta} \gamma/2$. The vertex without lines, $j = 0$, is considered as a connected diagram. In the following we subdivide the vertex factors into two parts, $ie^{\pm i\theta}\gamma/2$, related with vertex point, and $(\pm i)^j$, related with j corresponding outgoing lines. The propagators $\Delta(x) = \langle 0|T(\varphi(x)\varphi(0))|0\rangle$, where free field $\varphi(x)$ corresponds to the decomposition (45) of the Lagrangian, can be written as follows:

$$\Delta(x) = \int d^2k \ e^{ikx} \Delta(k), \qquad \Delta(k) = \frac{i}{\pi} \frac{1}{(k^2 - m^2 + i0)}, \tag{48}$$

where $d^2k = dk_0 dk_1$, $kx = k_0 x^0 + k_1 x^1$.

Any pair of vertices in diagrams can be connected by any number of propagators. As shown in the Appendix 1 (see also [32]) the perturbation theory can be reformulated in terms of sums over all such ways to connect a pair of vertices. We call these sums "superpropagators" (clearly this term has nothing to do with supersymmetry). For vertices of different type the superpropagator is equal to $e^{\Delta(x)}$, and for vertices of the same type it is $e^{-\Delta(x)}$. Due to the presence of the unity in the expansions of these exponents all "superdiagrams", constructed with superpropagators, are disconnected in conventional sense. In the following we hold to the conventional definition of the connectedness.

Consider the sum S'_n of all diagrams of order n in γ with fixed attachment of external lines and fixed types of vertices. This sum can be represented with one superdiagram, where each pair of vertices is connected by the superpropagator of corresponding type. Let S_n be the sum of only connected diagrams, contributing to the S'_n. We can write

$$S_n = S'_n - S''_n, \tag{49}$$

where the S''_n is corresponding sum of disconnected diagrams. The S''_n can be considered as a sum of terms which themselves are sums of all disconnected diagrams with fixed subdivision of the initial superdiagram into connected parts. Every such term is a product of some S_{n_1} with $n_1 < n$. Now one can write the decomposition (49) for every S_{n_1} and repeat the procedure up to the S_1, which is equal to the S'_1. Thus one can get the representation of the S_n in terms of a sum of products of the S'_j with $j \leq n$:

$$S_n = \sum \prod_k S'_{j_k}, \tag{50}$$

$$\sum_k j_k = n. \tag{51}$$

Let us use the x-representation. We estimate the index of UV divergency of the S_n via simple dimensional counting in the integrands, considering all vertex coordinates approaching each other (this estimation doesn't take into account possible UV divergences of subdiagrams [38]). Having in the integrand of the S_n a pole of order r'_n, one estimates the index as

$$2(n-1) - r'_n, \tag{52}$$

where $2(n-1)$ volume elements in the integrals are taken into account. To estimate the r'_n one can use the decomposition (50), applying it to the integrands of diagrams before integration.

Let us find the power r_j of the pole in the integrand of the S'_j. With this aim we represent the mentioned integrands in terms of superpropagators. Let a superdiagram S'_j has l_1 vertices of the 1st type and l_2 vertices of the 2nd type, $l_1 + l_2 = j$. Using the property of McDonald function $K_0(z)$ for $z \to 0$ we write

$$\Delta(x) \sim K_0(m\sqrt{x^2}) \sim \ln \frac{1}{x^2}. \tag{53}$$

Hence, corresponding superpropagators behave as follows:

$$e^{\pm\Delta(x)} \sim (x^2)^{\mp 1}. \tag{54}$$

At coinciding coordinates of vertices one can sum up the powers of x^2 for all superpropagators. The number of superpropagators joining vertices of the same type is $l_1(l_1-1)/2 + l_2(l_2-1)/2$, and the number of those joining vertices of different type is $l_1 l_2$. Therefore one can write

$$r_j = 2\left(l_1 l_2 - \frac{l_1(l_1-1)}{2} - \frac{l_2(l_2-1)}{2}\right) = j - (l_1 - l_2)^2 \le j. \tag{55}$$

Now the r'_n can be written as follows

$$r'_n = \max \sum_k r_{j_k}, \tag{56}$$

where the maximum is taken over the terms in the eq-n (50). According to eq-ns (51),(55) one gets

$$r'_n \le \max \sum_k j_k = n. \tag{57}$$

Hence for the index (52) we have

$$2(n-1) - r'_n \ge 2(n-1) - n = n - 2. \tag{58}$$

The necessary condition for the UV convergence of the S_n is the positivity of this index, i. e. the validity of the inequality $n > 2$. At $n = 1$ there is no UV divergency due to the normal ordering prescription in the Lagrangian. At $n = 2$ the eq-n (55) reads

$$r'_2 = r_2 = 2 - (l_1 - l_2)^2, \qquad l_1 + l_2 = 2, \tag{59}$$

hence, for corresponding index one has according to eq-n (52)

$$2(2-1) - r'_2 = (l_1 - l_2)^2. \tag{60}$$

The condition of positivity of this index is $l_1 = 2, l_2 = 0$ or $l_1 = 0, l_2 = 2$. This is fulfilled if both vertices are of the same type. For $l_1 = l_2 = 1$, i. e. for the vertices of different type, one may have logarithmic UV divergence for the S_2. However the complete contribution to the connected Green function in this

order is UV finite. Indeed, the corresponding Green function for the vertices of different type is (see Appendix 1)

$$G_N^{(2)}(y_1, \ldots, y_N) =$$
$$= -\left(\frac{\gamma}{2}\right)^2 \int d^2x_1 d^2x_2 \prod_{i=1}^N \left(\sum_{k_i=1,2} (-i)(-1)^{k_i} \Delta(y_i - x_{k_i})\right) e^{\Delta(x_1 - x_2)}, \qquad (61)$$

where the sum is over different ways of the attachment of external lines to the vertices. As $(x_1 - x_2) \to 0$ in the integrand, one has

$$\sum_{k_i=1,2} (-1)^{k_i} \Delta(y_i - x_{k_i}) \sim O(x_1 - x_2). \qquad (62)$$

Therefore divergent terms cancel in the integral (61), and corresponding Green function is UV finite for $N > 0$.

Let us consider the possibility that the S_n is UV divergent due to bad UV index of some subdiagrams [38]. Such subdiagrams can be only of 2nd order in γ with vertices of different type. The external lines of such subdiagrams can be either external or internal with respect to the whole diagram. To find the contribution of these subdiagrams to the S_n one has to sum over different ways to attach all lines external to the subdiagram. This gives the cancelation of divergences similar to eq-n (61). The cancelation takes place even for fixed attachment of external lines to the full diagram S_n, because the S_n is a connected diagram and therefore there is at least one internal line attached as external to each subdiagram and the summation over two ways to attach this line suffices for the cancelation.

Thus we have proved that the sum of all connected Lorentz-covariant diagrams of order n is UV finite, and that for $n \neq 2$ there is no UV divergencies even for fixed attachment of external lines, while for $n = 2$ one has to have nonzero number of external lines and to sum up over different ways of attachment of these external lines. Only vacuum diagrams of 2nd order and the sums S_2 with fixed attachment of external lines are UV divergent. Therefore all Green functions without vacuum loops are UV finite.

Let us remark that one cannot prove in similar way the UV finiteness of the LF perturbation theory because in this case some diagrams become equal to zero. This destroys the described cancelation of divergences.

4. The relation between LF and Lorentz-covariant superpropagators for finite x^-

Let us now consider the boson LF perturbation theory and compare it with Lorentz-covariant one. In the LF formulation the cutoff in LF momentum, $|p_-| \geq \varepsilon > 0$, is assumed, i. e. the Fourier modes of the field $\varphi(x)$ with $|p_-| < \varepsilon$ are excluded. It can be shown [39, 40] that the oldfashioned (noncovariant) perturbation theory, obtained with LF Hamiltonian, can be transformed to equivalent perturbation theory of Feynman type with the same integrands as in covariant diagrams, but in corresponding Feynman integrals the integration over p_+ is to be performed before the integration over p_- and the p_- integration has to be taken over the domain $|p_-| \geq \varepsilon > 0$. One can easily derive these facts formally via suitable transformations in functional integral [41]. Corresponding Feynman diagrams are called in the following LF diagrams.

In the paper [21] a method was proposed to find all diagrams (to all orders of perturbation theory) that give different results (in the $\varepsilon \to 0$ limit) when calculated in LF and Lorentz-covariant ways. Having these diagrams found one can add to canonical LF Hamiltonian counterterms, compensating the mentioned difference between LF and Lorentz-covariant perturbation theories. However straightforward application of the method of the paper [21] to the theory with Lagrangian (45) is impossible due to nonpolynomial type of the interaction term (this term generates infinite number of diagrams of a given order in γ, and some of partial infinite sums of these diagrams are UV divergent). We will overcome this difficulty in Sect. 6, truncating the series for the exponents in the interaction. After application of the method of the paper [21] we remove this truncation adding, if necessary, new counterterms to LF Hamiltonian to make the removing of the truncation correct. Let us find, first of all, the difference between the LF and Lorentz-covariant superpropagators.

Let us consider conventional covariant propagator (48), introducing UV regularization parameter Λ:

$$\Delta_\Lambda(x) = \frac{i}{\pi} \int_{-\Lambda}^{\Lambda} dk_1 \int dk_0 \frac{e^{ik_0 x^0 + ik_1 x^1}}{k_0^2 - k_1^2 - m^2 + i0}, \tag{63}$$

It can be rewritten in the following form (after integration over k_0):

$$\Delta_\Lambda(x) = \int_{-\Lambda}^{\Lambda} dk_1 \frac{e^{ik_1 x^1 - iE(k_1)|x^0|}}{E(k_1)}, \quad E(k_1) \equiv \sqrt{k_1^2 + m^2}. \tag{64}$$

Introducing new integration variable

$$k = \frac{1}{\sqrt{2}} \left(E(k_1) + k_1 \text{sign}(x^0) \right) \tag{65}$$

instead of k_1, one gets after simple transformations the following expression:

$$\Delta_\Lambda(x) = \int_{\varepsilon_\Lambda}^{V_\Lambda} \frac{dk}{k} \, e^{-i\left(kx^- + \frac{m^2}{2k}x^+\right)\text{sign}(x^0)}, \tag{66}$$

where

$$\varepsilon_\Lambda = \frac{1}{\sqrt{2}} \left(E(\Lambda) - \Lambda \right), \qquad V_\Lambda = \frac{1}{\sqrt{2}} \left(E(\Lambda) + \Lambda \right). \tag{67}$$

In the LF formulation the propagator, regularized by the cutoff $0 < \varepsilon \leq |p_-| \leq V$ has the form:

$$\Delta_{\varepsilon,V}^{lf}(x) = \frac{i}{\pi} \left(\int_{-V}^{-\varepsilon} + \int_{\varepsilon}^{V} \right) dk_- \int dk_+ \, \frac{e^{i(k_+ x^+ + k_- x^-)}}{2k_+ k_- - m^2 + i0}, \tag{68}$$

or, if $x^+ \neq 0$, after the integration over k_+,

$$\Delta_{\varepsilon,V}^{lf}(x) = \int_{\varepsilon}^{V} \frac{dk}{k} \, e^{-i\left(kx^- + \frac{m^2}{2k}x^+\right)\text{sign}(x^+)}. \tag{69}$$

At $x^2 \neq 0$ one can transform further these expressions taking the limit $\varepsilon_\Lambda, \varepsilon \to 0, V_\Lambda, V \to \infty$ and changing after that the integration variable $k \to k/|x^-|$:

$$\Delta_\Lambda(x) \to \Delta(x) = \int_0^\infty \frac{dk}{k} \, e^{-i\left(k + \frac{m^2}{4k}x^2\right)\text{sign}(x^0 x^-)}, \tag{70}$$

$$\Delta_{\varepsilon,V}^{lf}(x) \to \Delta^{lf}(x) = \int_0^\infty \frac{dk}{k} \, e^{-i\left(k + \frac{m^2}{4k}x^2\right)\text{sign}(x^2)}. \tag{71}$$

At $x^2 > 0$ one has $\text{sign}(x^0 x^-) = \text{sign}(x^2) = 1$. At $x^2 < 0$ one can show that $\Delta(x)$ and $\Delta^{lf}(x)$ are real (by looking at these integrals after the additional replacement of variable, $k \to -m^2 x^2/(4k)$), so that the sign of the argument of the exponent in eq-n (70) is irrelevant. Hence the $\Delta(x)$ and $\Delta^{lf}(x)$ can be written in the same form:

$$\Delta(x) = \Delta^{lf}(x) = \int_0^\infty \frac{dk}{k} \, e^{-i\left(k + \frac{m^2}{4k}x^2\right)}. \tag{72}$$

16

Therefore at $x^2 \neq 0$ the superpropagators $e^{\pm\Delta(x)}$ and $e^{\pm\Delta^{lf}(x)}$ also coincide. At $x^2 \to 0$ one has poles or powers in x^2 for these superpropagators according to eq-ns (53),(54). We need exact prescriptions for the poles of the superpropagators $e^{\Delta(x)}$ and $e^{\Delta^{lf}(x)}$ connecting the vertices of different types. For the superpropagator, connecting vertices of the same types, we get immediately:

$$e^{-\Delta(x)} = e^{-\Delta^{lf}(x)}. \tag{73}$$

For the Lorentz-covariant propagator we can apply at $x^2 \simeq 0, x^\mu \neq 0$ the following form:

$$\Delta(x) \sim -\ln\left(-\frac{m^2 e^{2C}}{4}(x^2 - i0)\right), \tag{74}$$

where C is Euler constant and the factor before the $(x^2 - i0)$ is the same as in the corresponding asymptotic decomposition of McDonald function K_0.

Hence we get at $x^2 \simeq 0, x^\mu \neq 0$

$$e^{\Delta(x)} \sim -\frac{4e^{-2C}}{m^2}\frac{1}{x^2 - i0}. \tag{75}$$

One can show that the prescription (75) is equivalent to the integration of the $e^{\Delta_\Lambda(x)}$ over x at first and then taking the $\Lambda \to \infty$ limit. The condition $x^\mu \neq 0$ is connected with the nonintegrability of the $e^{\Delta(x)}$ at $x^\mu = 0$. However the cutoff Λ plays, in fact, only intermediate role because all UV divergencies cancel in Green functions (as was shown in Sect. 3).

For the LF superpropagator we consider the domains $x^- \to 0, x^+ \neq 0$ and $x^+ \to 0, x^- \neq 0$ separately. In the domain $x^- \to 0, x^+ \neq 0$ we compare expressions (66) and (69) and notice that these expressions can be identified at $\varepsilon_\Lambda, \varepsilon \to 0, V_\Lambda, V \to \infty$ due to the equality $\text{sign}(x^+) = \text{sign}(x^0)$ in the considered domain. Therefore for the LF superpropagator we can use the same prescription (75) in this domain.

At $x^+ \to 0, x^- \neq 0$ we will consider the superpropagator $e^{\Delta^{lf}_{\varepsilon,V}(x)}$, integrated over x^+ at fixed ε, V, and prove that the result of the integration, taken in the $\varepsilon \to 0, V \to \infty$ limit, can be equivalently achieved via integration of LF superpropagator over x^+ with principle value prescription.

Consider small interval $|x^+| \leq \delta$. Outside this interval one can take for expressions (66) and (69) their $\varepsilon \to 0, V \to \infty$ limit and use the formula (75). Inside the interval $|x^+| \leq \delta$ we investigate the integral

$$I_\delta = \int\limits_{-\delta}^{\delta} dx^+ \, e^{\Delta^{lf}_{\varepsilon,V}(x)} \tag{76}$$

in the limit $\varepsilon \to 0, V \to \infty$. We have

$$I_\delta = \int_0^\delta dx^+ \left(e^{\Delta_{\varepsilon,V}^{lf}(x^+,x^-)} + e^{\Delta_{\varepsilon,V}^{lf}(-x^+,x^-)} \right) =$$

$$= \int_0^\delta dx^+ \ e^{\Delta_{\varepsilon,V}^{lf}(x)} \left(1 + e^{\Delta_{\varepsilon,V}^{lf}(-x^+,x^-)-\Delta_{\varepsilon,V}^{lf}(x^+,x^-)} \right), \tag{77}$$

where according to eq-n (69)

$$\Delta_{\varepsilon,V}^{lf}(-x^+,x^-) - \Delta_{\varepsilon,V}^{lf}(x^+,x^-) = 2i\text{sign}(x^+) \int_\varepsilon^V \frac{dk}{k} \ e^{-i\frac{m^2}{2k}|x^+|} \sin(kx^-) =$$

$$= 2i\text{sign}(x^+) \left(\int_\varepsilon^V \frac{dk}{k} \ \sin(kx^-) + \int_\varepsilon^V \frac{dk}{k} \ \sin(kx^-) \left(e^{-i\frac{m^2}{2k}|x^+|} - 1 \right) \right) =$$

$$= i\pi\text{sign}(x^2) + D(x) + O(\varepsilon) + O(V^{-1}), \tag{78}$$

where

$$D(x) = 2i\text{sign}(x^+) \int_0^\infty \frac{dk}{k} \ \sin(kx^-) \left(e^{-i\frac{m^2}{2k}|x^+|} - 1 \right) \tag{79}$$

and the quantities $O(\varepsilon)$ and $O(V^{-1})$ do not diverge at $x^+ \to 0$. Here we used the equality $\int_0^\infty dk \sin(kx^-)/k = \text{sign}(x^-)\pi/2$. So we obtain

$$I_\delta = \int_0^\delta dx^+ \ e^{\Delta_{\varepsilon,V}^{lf}(x)} \left(1 - e^{D(x)+O(\varepsilon)+O(V^{-1})} \right). \tag{80}$$

One can show, that at $x^+ \to 0$ the function $D(x)$ can be estimated as $|D(x)| \le c\sqrt{x^+}$. Let us remark that the function $e^{\Delta_{\varepsilon,V}^{lf}(x)}$ has a pole in x^+ regularized by the parameter ε, and, hence, at $|x^+| \le \varepsilon$ this function is bounded by a quantity of order $\ln(1/\varepsilon)$. Therefore the "divergency" at lower limit in the integral (80) is of the order $(O(\varepsilon) + O(V^{-1})) \ln \varepsilon$, which is negligible in the $V \to \infty, \varepsilon \to 0$ limit.

Thus we get the equality

$$\lim_{\delta \to 0} \lim_{\varepsilon \to 0} \lim_{V \to \infty} I_\delta = 0, \tag{81}$$

that can be considered as the evidence of the validity of the principle value prescription in x^+ for the LF superpropagator at $x^+ \sim 0$, $x^- \ne 0$.

Hence the analog of eq-n (75) for the LF superpropagator should have the form

$$e^{\Delta^{lf}(x)} \sim -\frac{4e^{-2C}}{m^2}\left(\mathcal{P}\frac{1}{x^+}\right)\frac{1}{2x^- - \text{sign}(x^+)i0}, \tag{82}$$

where the $\mathcal{P}$ denotes the principle value symbol. This is the limiting value of the superpropagator at $V \to \infty$, $\varepsilon \to 0$. It can not be used directly in the integrands of Feynman integrals because the superpropagator at finite ε and large x^- can differ essentially from its limiting value. The regularization $|p_-| \geq \varepsilon$ should be removed only after performing all integrations in Feynman integrals. In contrast with this, the regularization $|p_-| \leq V$ plays intermediate role and can be replaced by the prescription (82) at $x^- \approx 0$ (similarly to the situation with the cutoff Λ in Lorentz coordinates). This can be possible, in particular, because at finite ε the superpropagator $e^{\Delta^{lf}_\varepsilon(x)}$ is an integrable function at $x^\mu = 0$ in the limit $V \to \infty$ (we will use the notation $\Delta^{lf}_\varepsilon(x) = \lim_{V \to \infty} \Delta^{lf}_{\varepsilon,V}(x)$). The latter assertion can be verified by straightforward analysis of the asymptotic behavior at most singular points leading to the asymptotic behavior (75) in the domain $x^- \approx 0$, $x^+ \neq 0$, whereas the $x^+ \to 0$ singularity is regulated by the ε. This corresponds to well known fact that only ε-cutoff of LF momenta is sufficient for UV regularization of the LF perturbation theory in "longitudinal momenta" (because the magnitude of the external p_- momenta sets the upper bound for the internal ones). We will assume in the following that the cutoff $|p_-| \leq V$ is removed.

Taking into account that LF and Lorentz-covariant superpropagators coincide at $x^2 \neq 0$, and using (82), we can write for $x^\mu \neq 0$

$$e^{\Delta(x)} - e^{\Delta^{lf}(x)} = -\frac{4e^{-2C}}{m^2}\frac{1}{2x^- - \text{sign}(x^+)i0}\left(\frac{1}{x^+ - \text{sign}(x^-)i0} - \mathcal{P}\frac{1}{x^+}\right) =$$
$$= -\frac{2\pi i e^{-2C}}{m^2|x^-|}\delta(x^+). \tag{83}$$

The equalities (73) and (83) are valid for the limiting (at $\varepsilon \to 0$) values of the superpropagators $e^{\pm \Delta^{lf}_\varepsilon(x)}$. Therefore, these equalities can not be applied directly in LF diagrams, where one must perform all integrations before the limit $\varepsilon \to 0$ is taken. Nevertheless one can find in x-space a domain, depending on ε and having the following property: if the point x^μ stays in this domain when ε changes, then the mentioned equalities remain true up to terms vanishing in the $\varepsilon \to 0$ limit, even if one modifies the equalities by

replacing the limiting value $\Delta^{lf}(x)$ with the $\Delta^{lf}_\varepsilon(x)$. At $|x^+| \geq \beta > 0$, we can take the $\varepsilon \to 0$ limit in eq-n (69) if $\varepsilon/\beta \to 0$. In this domain the modified equalities (73),(83) are valid for all x^-. At $x^- \neq 0$ and at any x^+ we consider bounded domain $0 < \alpha \leq |x^-| \leq W$, and $W\varepsilon \to 0$ for $\varepsilon \to 0$. Due to this condition the integral

$$\int\limits_\varepsilon^\infty \frac{dk}{k} \ \sin(kx^-) = \text{sign}(x^-) \int\limits_{\varepsilon|x^-|}^\infty \frac{dk}{k} \ \sin k \tag{84}$$

contained in the eq-n (78) goes in the $\varepsilon \to 0$ limit to $\text{sign}(x^-)\pi/2$. Thus the modified equalities (73),(83) are valid in the domain

$$x^\mu \in \left\{0 < \beta \leq |x^+|\right\} \cup \left\{0 < \alpha \leq |x^-| \leq W\right\},$$
$$\frac{\varepsilon}{\beta} \xrightarrow[\varepsilon \to 0]{} 0, \qquad W\varepsilon \xrightarrow[\varepsilon \to 0]{} 0. \tag{85}$$

5. The construction of the counterterm generating the difference between Lorentz-covariant and LF superpropagators in the bounded domain of x^-

Let us consider the action, corresponding to the Lagrangian (45), in LF coordinates with the cutoff $|p_-| \geq \varepsilon > 0$ and with the additional counterterm of the form

$$\int d^2x d^2y \ : e^{i\varphi(x)} e^{-i\varphi(y)} : u(x - y), \tag{86}$$

where

$$u(x) = \tilde{u}(x^-)\delta(x^+). \tag{87}$$

and the $\tilde{u}(x^-)$ is a bounded function, rapidly decreasing at the infinity.

Analogously to the formula (A1.4) of the Appendix 1 we get for the N-point Green function

$$G_N(y_1, \ldots, y_N) = \sum_{k,l,m=0}^\infty \int \left(\prod_{j=1}^{k+l+2m} d^2x_j\right) \frac{(i\lambda)^k (i\bar{\lambda})^l \ i^m}{k!l!m!} \times$$
$$\times \prod_{j=1}^m \left(u(x_{k+j} - x_{k+l+m+j})e^{-\Delta^{lf}_\varepsilon(x_{k+j} - x_{k+l+m+j})}\right) h_{klm}, \tag{88}$$

where $\lambda = \frac{\gamma}{2}e^{i\theta}$, $\bar{\lambda} = \frac{\gamma}{2}e^{-i\theta}$,

$$h_{klm} = \left(\prod_{i,j=1;i<j}^{k+m} e^{-\Delta_\varepsilon^{lf}(x_i - x_j)} \right) \left(\prod_{i,j=1;i<j}^{l+m} e^{-\Delta_\varepsilon^{lf}(x_{i+k+m} - x_{j+k+m})} \right) \times$$

$$\times \left(\prod_{i=1}^{k+m} \prod_{j=1}^{l+m} e^{\Delta_\varepsilon^{lf}(x_i - x_{j+k+m})} \right) \times$$

$$\times \langle 0| T \prod_{i=1}^{N} \left(\varphi(y_i) + i \sum_{j=1}^{k+l+2m} \Delta_\varepsilon^{lf}(y_i - x_j) \text{sign}(k + m - j + \frac{1}{2}) \right) |0\rangle. \qquad (89)$$

and T is "LF time" ordering symbol.

Let us remark that $h_{klm} = h_{k+m,l+m,0}$. Introducing new summation indices $\tilde{k} = k + m$, $\tilde{l} = l + m$ we rewrite the formulae (88) in the form:

$$G_N(y_1, \ldots, y_N) = \sum_{\tilde{k}, \tilde{l}=0}^{\infty} \int \left(\prod_{j=1}^{\tilde{k}+\tilde{l}} d^2 x_j \right) \frac{(i\lambda)^{\tilde{k}} (i\bar{\lambda})^{\tilde{l}}}{\tilde{k}! \tilde{l}!} \sum_{m=0}^{\min(\tilde{k},\tilde{l})} C_{\tilde{k}}^m C_{\tilde{l}}^m m! \times$$

$$\times \prod_{j=1}^{m} \left(\frac{-i}{|\lambda|^2} u(x_{\tilde{k}-j+1} - x_{\tilde{k}+\tilde{l}-j+1}) e^{-\Delta_\varepsilon^{lf}(x_{\tilde{k}-j+1} - x_{\tilde{k}+\tilde{l}-j+1})} \right) h_{\tilde{k}\tilde{l}0}, \qquad (90)$$

where $C_n^m = \frac{n!}{m!(n-m)!}$.

We transform further this expression, taking into account the locality in time of the $u(x)$, eq-n (87), and the estimation $e^{-\Delta_\varepsilon^{lf}(x)}|_{x^+=0} = O(\varepsilon)$ which follows from the eq-n (69), so that, if contained in the integrand,

$$u(x_i - x_j) u(x_i - x_k) e^{-\Delta_\varepsilon^{lf}(x_j - x_k)} = O(\varepsilon). \qquad (91)$$

This enables us to rewrite the eq-n (90) in the form

$$G_N(y_1, \ldots, y_N) = \sum_{\tilde{k}, \tilde{l}=0}^{\infty} \int \left(\prod_{j=1}^{\tilde{k}+\tilde{l}} d^2 x_j \right) \frac{(i\lambda)^{\tilde{k}} (i\bar{\lambda})^{\tilde{l}}}{\tilde{k}! \tilde{l}!} \times$$

$$\times \prod_{i=1}^{\tilde{k}} \prod_{j=1}^{\tilde{l}} \left(1 - \frac{i}{|\lambda|^2} u(x_i - x_{\tilde{k}+j}) e^{-\Delta_\varepsilon^{lf}(x_i - x_{\tilde{k}+j})} \right) h_{\tilde{k}\tilde{l}0} + O(\varepsilon), \qquad (92)$$

that can be checked using eq-n (91), the symmetry of the $h_{\tilde{k}\tilde{l}0}$ under the permutation of variables $x_1, \ldots, x_{\tilde{k}}$ and of the variables $x_{\tilde{k}+1}, \ldots, x_{\tilde{k}+\tilde{l}}$, as well as the possibility to replace these variables in the integrals of the eq-n (92).

Let us remark that the eq-n (92) resembles the expression for the Green function (A1.4) for the theory without counterterms if the superpropagator $e^{\Delta_\varepsilon^{lf}(x)}$ is replaced by

$$e^{\Delta_\varepsilon^{lf}(x)} \left(1 - \frac{i}{|\lambda|^2} u(x) e^{-\Delta_\varepsilon^{lf}(x)} \right) = e^{\Delta_\varepsilon^{lf}(x)} - \frac{4i}{\gamma^2} \tilde{u}(x^-) \delta(x^+). \qquad (93)$$

According to eq-n (83) we can choose the form of the $\tilde{u}(x^-)$ so that the expression (93) coincides in the domain (85) with Lorentz-covariant superpropagator $e^{\Delta(x)}$ in the $\varepsilon \to 0$ limit. This form is

$$\tilde{u}(x^-) = \frac{\pi}{2} e^{-2C} \frac{\gamma^2}{m^2 |x^-|} \theta(|x^-| - \alpha) v(\varepsilon x^-), \tag{94}$$

where the factor $\theta(|x^-| - \alpha)$ with $\alpha > 0$ is introduced to regularize the pole in x^-. Furthermore for generality an arbitrary continuous function $v(\varepsilon x^-)$, rapidly decreasing at infinity and satisfying the condition $v(0) = 1$, is included. The decrease of the function $v(\varepsilon x^-)$ guarantees that the function $\tilde{u}(x^-)$ also decreases as it was required earlier (see the text after eq-n (87)). The functions, introduced above, can influence the function $\tilde{u}(x^-)$ only outside of the domain (85).

Now we can get the expression (93) via addition of the quantity

$$-\frac{2\pi i}{m^2} e^{-2C} \delta(x^+) \frac{\theta(|x^-| - \alpha)}{|x^-|} v(\varepsilon x^-) \tag{95}$$

to the LF superpropagator $e^{\Delta^{lf}_\varepsilon (x)}$.

Thus the LF theory with the counterterm

$$
\begin{aligned}
S_c &= \frac{\pi}{2} e^{-2C} \frac{\gamma^2}{m^2} \int d^2x d^2y \left(: e^{i\varphi(x)} e^{-i\varphi(y)} : -1 \right) \times \\
&\quad \times \delta(x^+ - y^+) \theta(|x^- - y^-| - \alpha) \frac{v(\varepsilon(x^- - y^-))}{|x^- - y^-|} = \\
&= \frac{\pi}{2} e^{-2C} \frac{\gamma^2}{m^2} \int dx^+ \left(\int dx^- dy^- \left(: e^{i\varphi(x^+, x^-)} e^{-i\varphi(x^+, y^-)} : -1 \right) \times \right. \\
&\quad \left. \times \theta(|x^- - y^-| - \alpha) \frac{v(\varepsilon(x^- - y^-))}{|x^- - y^-|} \right)
\end{aligned}
\tag{96}
$$

can be reformulated as a theory with superpropagator which is equal to a Lorentz-covariant one in the domain (85), but, perhaps, differs from it outside this domain. In eq-n (96) we subtract the unity from the product of the exponents (that is irrelevant for physical results, because one gets only additional constant in the action). Besides, we remove the δ-function integrating over y^+. One can see now, that the addition to the action of local in time (but not in the x^-) counterterm S_c is equivalent to the addition of corresponding counterterm to the LF Hamiltonian. Let us remark also, that the expression (96) must be real. So we impose the additional condition $v^*(z) = v(-z)$ on the function $v(z)$, introduced above.

The compensation of the difference between LF and Lorentz-covariant superpropagators at finite x^- (i. e. in the bounded domain (85)), what was done in this and previous sections, is not needed for usual theories with polynomial interaction, because in such theories the mentioned difference is absent in the limit $\varepsilon \to 0$. The domain of unbounded x^- plays also an essential role. This domain corresponds, in a sense, to the vicinity of the point $p_- = 0$. Therefore, this domain is relevant for the estimation of difference between LF and Lorentz-covariant perturbation theory even for the theories with polynomial interaction. In the next section we will show how to correct the whole LF perturbation theory to make it equivalent with Lorentz-covariant one, using the compensation method described above and taking into account the domain of unbounded x^-.

6. Complete comparison of LF and Lorentz-covariant perturbation theories

Consider a perturbation theory for the Green functions without vacuum loops, generated by LF Hamiltonian, which is deduced from the action

$$S = \int d^2x \left(\frac{1}{8\pi} \left(\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2 \right) + B : e^{i\varphi} : + B^* : e^{-i\varphi} : \right) +$$
$$+ \frac{2\pi}{m^2} e^{-2C} |B|^2 \int d^2x d^2y \left(: e^{i\varphi(x)} e^{-i\varphi(y)} : -1 \right) \times$$
$$\times \delta(x^+ - y^+) \theta(|x^- - y^-| - \alpha) \frac{v(\varepsilon(x^- - y^-))}{|x^- - y^-|} \tag{97}$$

by canonical LF quantization. We will show that this perturbation theory is equivalent in the limit $\alpha \to 0$, $\varepsilon \to 0$ to usual Lorentz-covariant perturbation theory, corresponding to the Lagrangian (45). The formula (97) contains some complex quantity B, which is a function of the parameters γ, θ, α and ε. Perturbatively it can be decomposed as follows:

$$B = \frac{\gamma}{2} e^{i\theta} + \sum_{k=2}^{\infty} B_k \gamma^k. \tag{98}$$

Notice that in the expression (97) the term linear in B coincides in lowest (1st) order in γ with the interaction term (47) of the Lagrangian. The term in the action (97), quadratic in B, coincides in lowest (2nd) order with the counterterm (96).

23

Let us start with diagrams of 2nd order in γ corresponding to Lagrangian (45) (Lorentz-covariant and LF results obviously coincide in the 1st the order). Disconnected diagrams of 2nd order can be considered similar to the 1st order ones. Connected diagrams can differ by the type of vertices and by the type of attachment of external lines. Let us at first consider connected diagrams with two vertices of the same type. The sum of all diagrams of this kind with fixed attachment of external lines is UV finite. If all external lines are attached only to one vertex (from now on we call these diagrams "generalized tadpole diagrams", independently of their order), the diagrams are similar to vacuum ones, and LF result for these diagrams is zero. But in Lorentz-covariant theory the corresponding result is nonzero. Hence, to correct LF Hamiltonian corresponding to the Lagrangian (45) one should add a counterterm of second order, generating the results, $\tilde{S}_2^{1,1}$ and $\tilde{S}_2^{2,2}$, of Lorentz-covariant calculation of the sums of all such diagrams (with both vertices of the 1st or both vertices of the 2nd type, respectively):

$$\tilde{S}_2^{1,1} = -\frac{\gamma^2}{4} e^{2i\theta} \int d^2x \left(e^{-\Delta(x)} - 1 \right),$$
$$\tilde{S}_2^{2,2} = -\frac{\gamma^2}{4} e^{-2i\theta} \int d^2x \left(e^{-\Delta(x)} - 1 \right). \tag{99}$$

If the vertices are of the same type and external lines are attached to both of them, we can write for the sum of these diagrams in Lorentz-covariant theory the following expression (up to some vertex factors):

$$S_2^- = \gamma^2 \int d^2x \left(e^{-\Delta(x)} - 1 \right) e^{ipx}, \tag{100}$$

where p is the total external momentum going through these diagrams, $px = p_+ x^+ + p_- x^-$. We assume that the values of external momenta are "non-special". This means that any partial sum of "minus" components of these momenta is not equal to zero. The coincidence of Green functions at non-special values of external momenta is enough for their physical equivalence, because all special values of external momenta form a set of zero measure, and a variation of Green functions on this set does not change physical results. So we will consider all diagrams only at nonspecial values of external momenta. Let us rewrite the eq-n (100) in the form

$$S_2^- = \gamma^2 \int d^2x \lim_{m \to \infty} \sum_{m'=1}^{m} \frac{1}{m'!} \left(-\Delta(x) \right)^{m'} e^{ipx}, \tag{101}$$

and change the order of the integration and of the limiting procedure (it is possible owing to uniform convergence in m of the integral, what can be proved, using the fact that the limit of the integrand is a continuous function):

$$S_2^- = \gamma^2 \lim_{m \to \infty} \int d^2x \sum_{m'=1}^{m} \frac{1}{m'!} \left(-\Delta(x)\right)^{m'} e^{ipx}. \tag{102}$$

For any fixed finite m we can relate this quantity with a theory of polynomial type (with finite number of diagrams in each order) and apply the results of the paper [21] (see also Appendix 2). So one can show that for all diagrams with nonspecial values of external momenta, excepting diagrams which are (or contain as a subdiagram) generalized tadpole, the results of LF and Lorentz-covariant perturbation theories coincide in the $\varepsilon \to 0$ limit. So in the eq-n (102) (where the exceptional momenta mentioned above do not contribute) one can replace Lorentz-covariant propagator with LF one, taking the limit which removes LF cutoff parameters:

$$S_2^- = \gamma^2 \lim_{m \to \infty} \lim_{\varepsilon \to 0} \int d^2x \sum_{m'=1}^{m} \frac{1}{m'!} \left(-\Delta_\varepsilon^{lf}(x)\right)^{m'} e^{ipx}. \tag{103}$$

Using the fact that the sum of the series in eq-n (103) converges in the limit $\varepsilon \to 0$ to continuous function (i. e. to the $e^{-\Delta^{lf}(x)}$) one can prove that the integration over x and the ε limit converge uniformly with respect to m. Hence we can again change the order of the limiting procedures and of the integration and write:

$$S_2^- = \gamma^2 \lim_{\varepsilon \to 0} \int d^2x \left(e^{-\Delta_\varepsilon^{lf}(x)} - 1\right) e^{ipx}. \tag{104}$$

Thus Lorentz-covariant and LF results for the considered sum of diagrams coincide in the limit $\varepsilon \to 0$.

For connected Lorentz-covariant diagrams of the 2nd order with vertices of different type and with fixed attachment of external lines we may have UV divergent sums of diagrams. These sums are related with corresponding superpropagators which have singular behavior at $x^\mu \to 0$. Owing to UV finiteness of Green functions, proved above in Sect. 3, we can introduce any intermediate UV regularization for the Lorentz-covariant propagator to define corresponding superpropagator at $x^\mu \to 0$. Here we choose for convenience the following one:

$$\Delta_{reg}(x) = \begin{cases} \Delta_\varepsilon^{lf}(x), & \{|x^-| \le \alpha\} \cap \{|x^+| \le \beta\} \\ \Delta(x), & \{|x^-| > \alpha\} \cup \{|x^+| > \beta\} \end{cases}, \tag{105}$$

where α and β are cutoff parameters. As was remarked already (see the end of Sect. 4) the superpropagator $e^{\Delta^{lf}_\varepsilon(x)}$ is integrable function at $x^\mu = 0$ and fixed $\varepsilon > 0$. Thus we get the UV regularization, which can be removed in the $\alpha \to 0$, $\beta \to 0$ limit.

Let us consider connected 2nd order diagrams with all external lines attached to only one vertex. These diagrams are generalized tadpoles. The sum of all these diagrams in Lorentz-covariant theory is described by the expression:

$$\tilde{S}_2^{1,2} = \tilde{S}_2^{2,1} = -\frac{\gamma^2}{4} \int d^2x \left(e^{\Delta_{reg}(x)} - 1 \right), \qquad (106)$$

where the $\tilde{S}_2^{1,2}$ and $\tilde{S}_2^{2,1}$ are sums of diagrams with external lines attached to the vertices of the 1st and of the 2nd type correspondingly. Due to the convergence of this integral at large x one can rewrite it as

$$\tilde{S}_2^{1,2} = \tilde{S}_2^{2,1} = -\frac{\gamma^2}{4} \int\limits_{-W}^{W} dx^- \int dx^+ \left(e^{\Delta_{reg}(x)} - 1 \right) + \xi, \qquad (107)$$

where ξ denotes the quantity vanishing at $W \to \infty$. For the x in the domain (85) we use the formula (83), where we write $\Delta^{lf}_\varepsilon(x)$ instead of $\Delta^{lf}(x)$ according to the remark after the formula (83). Furthermore we can introduce the factor $v(\varepsilon x^-)$ as in the formula (94), because this factor goes to unity in the $W\varepsilon \to 0$ limit. We get in this way

$$\tilde{S}_2^{1,2} = \tilde{S}_2^{2,1} = -\frac{\gamma^2}{4} \times$$

$$\times \int\limits_{-W}^{W} dx^- \left(\int dx^+ \left(e^{\Delta^{lf}_\varepsilon(x)} - 1 \right) - 2\pi i e^{-2C} \frac{\theta(|x^-| - \alpha)}{m^2 |x^-|} v(\varepsilon x^-) \right) + \tilde{\xi}, \qquad (108)$$

where the definition (105) of the propagator in the domain $|x^+| \leq \beta$, $|x^-| \leq \alpha$ is taken into account. Here the $\tilde{\xi}$ denotes the quantity vanishing in the $\varepsilon \to 0$ limit which is taken at first, and only then the $W \to \infty$ limit. Let us rewrite eq-n (108) in the form:

$$\tilde{S}_2^{1,2} = \tilde{S}_2^{2,1} = -\frac{\gamma^2}{4} \int d^2x \left(e^{\Delta^{lf}_\varepsilon(x)} - 1 \right) + \frac{i\pi\gamma^2 e^{-2C}}{2m^2} \int dx^- \frac{\theta(|x^-| - \alpha)}{|x^-|} v(\varepsilon x^-) +$$

$$+ \frac{\gamma^2}{4} \int\limits_{|x^-| > \varepsilon W} dx^- \left(\int dx^+ \left(e^{\Delta^{lf}_1(x)} - 1 \right) - 2\pi i e^{-2C} \frac{v(x^-)}{m^2 |x^-|} \right) + \tilde{\xi}, \qquad (109)$$

where $\Delta_1^{lf}(x) \equiv \Delta_\varepsilon^{lf}(x)\mid_{\varepsilon=1}$. Here in the third term we have rechanged the integration variables $x^- \to x^-/\varepsilon$, $x^+ \to \varepsilon x^+$. The first term in this formula equals to zero, because it is LF generalized tadpole diagram (this corresponds to known fact that such LF diagrams as well as vacuum LF diagrams are equal to zero when the cutoff $|p_-| \geq \varepsilon > 0$ is assumed). The second term can be generated by the counterterm (96) (see Sect. 5), if one adds corresponding counterterm to canonical LF Hamiltonian. Let us assume that we have done this. The third term can not diverge in the $W\varepsilon \to 0$ limit, because the variation of this term with respect to variation of W is very small (two first terms and whole expression $\tilde{S}_2^{1,2}$ do not depend on W, and the expression $\tilde{\xi}$ depends weakly on the W). Therefore we can write the third term in the $W\varepsilon \to 0$ limit as some finite complex constant:

$$\frac{\gamma^2}{4} \lim_{s \to 0} \int\limits_{|x^-|>s} dx^- \left(\int dx^+ \left(e^{\Delta_1^{lf}(x)} - 1 \right) - 2\pi i e^{-2C} \frac{v(x^-)}{m^2|x^-|} \right). \tag{110}$$

The contribution related with this constant also should be generated by a new counterterm of second order. In the following it will be useful to write the expression (110) (up to a value vanishing in the $\varepsilon \to 0$ limit) in the form of the difference between the $\tilde{S}_2^{1,2}$ and the 2nd term in the eq-n (109), i. e. as the sum of corresponding 2nd order Lorentz-covariant generalized tadpoles and of corresponding contribution generated by the counterterm (96) but taken with the minus sign. Last term will be considered as one compensating the direct contribution of the counterterm (96) to the diagram of generalized tadpole type in the 2nd order. This direct contribution (which is the same for diagrams with all external lines attached to the vertex of the 1st type and to the vertex of the 2nd type) can be written as $i\frac{\gamma^2}{4}w$, where

$$w = \frac{2\pi e^{-2C}}{m^2} \int dx^- \frac{\theta(|x^-| - \varepsilon\alpha)}{|x^-|} v(x^-). \tag{111}$$

Let us consider now the sum of all connected diagrams of the 2nd order with vertices of different type and with external lines attached to both vertices. This sum can be described in Lorentz-covariant theory as follows (up to some vertex factors):

$$S_2^+ = \gamma^2 \int d^2x \left(e^{\Delta_{reg}(x)} - 1 \right) e^{ipx} \tag{112}$$

where p is total external momentum going through these diagrams and $\Delta_{reg}(x)$

is defined by the eq-n (105). Similar to previous case, we get:

$$S_2^+ = \int d^2x \left(e^{\Delta_\varepsilon^{lf}(x)} - 1 \right) e^{ipx} - 2\pi i e^{-2C} \int dx^- \frac{\theta(|x^-| - \alpha)}{m^2 |x^-|} v(\varepsilon x^-) e^{ip_- x^-} -$$

$$- \int\limits_{|x^-| > \varepsilon W} dx^- \left(\int dx^+ \left(e^{\Delta_1^{lf}(x)} - 1 \right) e^{ip_+ x^+} - 2\pi i e^{-2C} \frac{v(x^-)}{m^2 |x^-|} \right) e^{ip_- x^-/\varepsilon} + \tilde{\xi}. \qquad (113)$$

The first term of this expression coincides with LF calculation. The second term can be generated by the counterterm (96). The third term goes to zero in the $\varepsilon \to 0$ limit at $p_- \neq 0$ owing to the convergence of the corresponding integral at the $x^- = 0$ (as explained above). Thus Lorentz-covariant and LF results for the considered sum of diagrams coincide (for nonexceptional p_-).

Thus to make LF and Lorentz-covariant theories perturbatively equivalent to 2nd order in γ we have to add to canonical LF Hamiltonian the counterterm corresponding to the expression (96), and also counterterms compensating the contribution of the counterterm (96) to diagrams of generalized tadpole type in the 2nd order, and furthermore the counterterms, generating the expressions $\tilde{S}_2^{1,1}$, $\tilde{S}_2^{2,2}$, $\tilde{S}_2^{1,2}$ and $\tilde{S}_2^{2,1}$. It is easy to show, that the sum of all these counterterms (excepting the first one) is given by the expression

$$- i \sum_{l=0}^\infty \frac{: (i\varphi)^l :}{l!} \left(\tilde{S}_2^{1,1} + \tilde{S}_2^{1,2} - i\frac{\gamma^2}{4} w \right) = \left(-i\tilde{S}_2^{1,1} - i\tilde{S}_2^{1,2} - \frac{\gamma^2}{4} w \right) : e^{i\varphi} : \qquad (114)$$

(which corresponds the attachment of external lines to the vertex of the 1st type in the generalized tadpole diagram) and by the expression

$$- i \sum_{l=0}^\infty \frac{: (-i\varphi)^l :}{l!} \left(\tilde{S}_2^{2,2} + \tilde{S}_2^{2,1} - i\frac{\gamma^2}{4} w \right) =$$

$$= \left(-i\tilde{S}_2^{2,2} - i\tilde{S}_2^{2,1} - \frac{\gamma^2}{4} w \right) : e^{-i\varphi} :, \qquad (115)$$

(which corresponds to the attachment of lines to the vertex of the 2nd type). Here we have added nonessential constant at $l = 0$ for convenience. Using the Wick rotation one can show that the quantities

$$\int d^2x \left(e^{\pm \Delta(x)} - 1 \right) \qquad (116)$$

are imaginary, and, hence, the $\tilde{S}_2^{1,2}$ and the $\tilde{S}_2^{2,1}$ are imaginary, and $\tilde{S}_2^{2,2} = -\tilde{S}_2^{1,1*}$. Let us introduce the notation

$$A_2 = \frac{1}{\gamma^2} \left(-i\tilde{S}_2^{1,1} - i\tilde{S}_2^{1,2} \right) =$$

$$= \frac{i}{4} e^{2i\theta} \int d^2x \left(e^{-\Delta(x)} - 1 \right) + \frac{i}{4} \int d^2x \left(e^{\Delta_{reg}(x)} - 1 \right) \tag{117}$$

such that the $A_2 \gamma^2$ is the sum of connected Lorentz-covariant generalized tadpole diagrams of the 2nd order with external lines, attached to the vertex of the 1st type (these diagrams should be calculated with the help of the regularization (105)). We remark that

$$- i\tilde{S}_2^{2,2} - i\tilde{S}_2^{2,1} - \frac{\gamma^2}{4} w = \left(-i\tilde{S}_2^{1,1} - i\tilde{S}_2^{1,2} - \frac{\gamma^2}{4} w \right)^*, \tag{118}$$

and conclude that the action (97) contains all necessary counterterms of the 2nd order in γ, if one takes

$$B_2 = A_2 - \frac{1}{4} w. \tag{119}$$

This quantity is finite in the $\varepsilon \to 0$, $\alpha \to 0$ limit due to the finiteness of the quantity (110) and of the $\tilde{S}_2^{1,1}$ (despite of the divergency of the $w = w(\varepsilon\alpha)$ at $\varepsilon\alpha \to 0$).

Thus in the 2nd order in γ the LF perturbation theory, generated by the action (97) at some finite B_2, coincides in the $\varepsilon \to 0$, $\alpha \to 0$ limit with the Lorentz-covariant perturbation theory, generated by the Lagrangian (45).

Notice, that the coincidence of LF and Lorentz-covariant expressions for the sum of all connected diagrams of second order in γ, which are not generalized tadpoles, was proved only for such values of external momentum p, going through the diagram, that $p_- \neq 0$ (see, in particular, after eq-n (113)). It is enough for the coincidence of Green functions in the second order because the domain $p_- = 0$ is a set of zero measure. But the corresponding coincidence at any x in coordinate space is not guaranteed in principle. This remark can be important for the x space analysis of higher order diagrams, which contain the 2nd order diagrams as subdiagrams. As we will show in sequel, it suffices for this analysis that the LF and Lorentz-covariant expressions in coordinate representation coincide for 2nd order subdiagrams which are not generalized tadpole and have vertices of different type. Furthermore, this coincidence is needed only in the domain where the coordinates x_1^+, x_2^+ of these vertices are not equal to the corresponding coordinates of the external points (with respect to the subdiagram). Let us prove that this coincidence has place.

Going from diagrams in p-space to ones in x-space one has to add factors, corresponding to propagators of external lines (instead of exponents, containing external momenta). After the transition to the x-space all reasonings used

after the formula (112) can be repeated, with the exception of the estimation of the 3d term in the eq-n (113), where the condition $p_- \neq 0$ was used. The analog of this term in the x space can be written as follows (for simplicity we consider only two external points, y_1 and y_2):

$$- \int d^2 x_1 d^2 x_2 \Delta_\varepsilon^{lf}(y_1 - x_1) \theta(|x_1^- - x_2^-| - W) \times$$

$$\times \left(e^{\Delta_\varepsilon^{lf}(x_1 - x_2)} - 1 - 2\pi i \delta(x_1^+ - x_2^+) e^{-2C} \frac{v(\varepsilon(x_1^- - x_2^-))}{m^2 |x_1^- - x_2^-|} \right) \Delta_\varepsilon^{lf}(x_2 - y_2). \quad (120)$$

The propagator $\Delta_\varepsilon^{lf}(z)$ diminishes as $z^- \to \infty$ at $z^+ \neq 0$. Hence, if $x_1^+ \neq y_1^+$ and $x_2^+ \neq y_2^+$, at least one of the factors $\Delta_\varepsilon^{lf}(y_1 - x_1)$, $\Delta_\varepsilon^{lf}(x_2 - y_2)$ diminish due to the finiteness of y_1^- and y_2^- and due to the inequality $|x_1^- - x_2^-| \geq W$. Therefore the quantity (120) goes to zero in the $\varepsilon \to 0$, $W \to \infty$ limit.

Let us now prove that in the $\varepsilon \to 0$, $\alpha \to 0$ limit the LF perturbation theory, generated by the action (97), coincides with the Lorentz-covariant perturbation theory, generated by the Lagrangian (45), to all orders in γ at some choice of the quantities B_n.

We consider the sum S_n of all connected Lorentz-covariant diagrams of order $n > 2$ with fixed attachment of external lines, i. e. UV finite quantity, introduced above in Sect. 3. It is convenient to represent this quantity as a sum of special classes of diagrams.

The S_n can be described in terms of "incomplete superpropagators" $(e^{\pm\Delta(x)} - 1)$. The S_n is a sum of terms, corresponding to connection or disconnection of each pair of vertices by incomplete superpropagator. Let us rewrite the incomplete superpropagators as limits of finite sums $D_\pm^m(x)$ (they will be called "cutoff incomplete superpropagators"):

$$e^{\pm\Delta(x)} - 1 = \lim_{m \to \infty} D_\pm^m(x) = \lim_{m \to \infty} \sum_{m'=1}^{m} \frac{1}{m'!} (\pm\Delta(x))^{m'}. \quad (121)$$

One can write the S_n symbolically as follows

$$S_n = \int d^2 x_1 \dots d^2 x_n \times$$

$$\times \lim_{m_1,\dots \to \infty} \sum_d \left(D_+^{m_1}(x_{k_1} - x_{l_1}) \times \dots \times D_-^{m_2}(x_{k_2} - x_{l_2}) \times \dots \right), \quad (122)$$

where the $\sum_d$ denotes the summation over different ways to connect or disconnect the pairs of vertices by incomplete superpropagators. According to the definition of the S_n there are no disconnected diagrams among the terms of this sum, but there are terms which include the generalized tadpole subdiagrams. There is also possible that the S_n is a sum of generalized tadpole

30

diagrams. The generalized tadpole diagrams will play essential role in the following analysis.

Let us investigate the S_n in Lorentz-covariant perturbation theory. One can prove, that the functions $D_-^m(x)$ are integrable uniformly with respect to m, because they tend to continuous function at $m \to \infty$. To prove the uniform (w.r. to m) integrability of the $D_+^m(x)$ we can use the behavior (75) of the superpropagator at $x^2 \to 0, x^\mu \neq 0$, that allows to remove the integration contours apart from the points with $x^2 = 0$. Furthermore, we can take into account the cancellation of UV divergencies at $x^\mu = 0$, as shown in Sect. 3 in the proof of the UV finiteness. This allows to prove that the integrability is uniform in m in this case also. Therefore one can change the order of the integration and the limiting procedures in the eq-n (122):

$$S_n = \lim_{m_1,\ldots\to\infty} \int d^2x_1 \ldots d^2x_n \times$$
$$\times \sum_d \left(D_+^{m_1}(x_{k_1} - x_{l_1}) \times \ldots \times D_-^{m_2}(x_{k_2} - x_{l_2}) \times \ldots \right). \tag{123}$$

Let us remark that the $\sum_d$ in this equation can be interchanged with the integration (owing to UV finiteness of any finite sum of diagrams), but it can not be interchanged with the limits in m_i, because, as was shown in Sect. 3, only the total sum $\sum_d$ is UV finite in this limit (not separate terms).

At finite m_i one can apply the method of the paper [21] to compare the results of LF and Lorentz-covariant calculations of these quantities. One concludes (see also Appendix 2) that the coincidence takes place in the limit $\varepsilon \to 0$ in all cases with the exception of generalized tadpole diagrams and diagrams including them as subdiagrams. These generalized tadpole subdiagrams enter as factors before the rest part of the diagram, and therefore we can apply the method of the paper [21] only to these rest parts. Then we can replace in the expression (123) Lorentz-covariant propagators with LF ones if these propagators do not enter in generalized tadpole subdiagrams. In Lorentz-covariant propagators contained in generalized tadpole diagrams it is convenient to introduce the regularization (105) (let us remind, that LF value of generalized tadpole diagrams is always zero). Thus we can rewrite the symbolic expression (123) in the form

$$S_n = \lim_{m_1,\ldots\to\infty} \lim_{\alpha\to0} \lim_{\varepsilon\to0} \int d^2x_1 \ldots d^2x_n \sum_d \left(D_{\varepsilon+}^{lf\,m_1}(x_{k_1} - x_{l_1}) \times \ldots \times \right.$$
$$\times D_+^{m_2}(x_{k_2} - x_{l_2}) \times \ldots \times D_{\varepsilon-}^{lf\,m_3}(x_{k_3} - x_{l_3}) \times \ldots \times D_-^{m_4}(x_{k_4} - x_{l_4}) \times \ldots \Big). \tag{124}$$

where the ε is LF cutoff parameter, used for the LF propagators, and the α is the parameter of the regularization (105) of Lorentz-covariant propagators, participating in generalized tadpole subdiagrams and contained in $D_{\pm}^{m_i}$ in the eq-n (124).

If we could move the limits in m_i through the limits in ε, α and through the operation of integration, we could write incomplete superpropagators instead of cutoff incomplete ones and reproduce the Lorentz-covariant result for the S_n by means of the canonical LF Hamiltonian with counterterms generating the contribution equal to that of Lorentz-covariant generalized tadpole diagrams. However to do this we need uniform convergence in m_i of the integrals and of the limits in ε, α. This convergence depends on the behavior of the functions $D_{\varepsilon\pm}^{lf\,m}(x)$ in the integrand. Repeating the reasonings given before the formula (123), we conclude that there is a difficulty only with the $D_{\varepsilon+}^{lf\,m}(x)$ at $x^+ = 0$, $x^- \neq 0$, because LF propagator is nonanalytical function in x^+ at $x^+ = 0$ (see eq-n (69)), and therefore it is not allowed to remove the integration contour apart from this singular point. However we can take into account that the integrand contains the products of several cutoff superpropagators. For any 3 vertices one has at least one superpropagator corresponding to vertices of the same type. Therefore if the x^+ coordinates of all these 3 vertices tend to coincide, the behavior of the integrand at singular point becomes integrable, because we have for the superpropagator, connecting the vertices of the same type, the estimation

$$e^{-\Delta_\varepsilon^{lf}(x)} \sim c_1 x^+ x^- + c_2 \varepsilon x^-, \qquad x^+ \to 0, \qquad (125)$$

what improves the convergence in x^+ (let us remark that the ε regularizes the singularity at $x^+ = 0$ and that it is expected that the domain $x^- \to \infty$ does not play essential role). This allows to prove that the contribution of considered integration domain vanishes. If the x^+ coordinates of two vertices of different type tend to coincide and are not equal to x^+ coordinate of some 3d vertex, we can not prove the uniform convergence of the integrals in this domain. In general case the integration domain can be subdivided into domains of two classes. We define these classes in such a way that in the first class the proof of the uniform convergence fails (i. e. the x^+ coordinates of some of pairs of vertices of different type coincide, but are not equal to the x^+ coordinates of other vertices), and in the second class the uniform convergence can be proved. For the second class we can come back to incomplete superpropagators from cutoff incomplete ones in the integrand of the eq-n (124).

For the first class we do the following: (1) we replace in reverse order the cutoff incomplete LF superpropagators by corresponding Lorentz-covariant ones (justifying this step as above by referring to the paper [21]); (2) then we use the uniform convergence property proved for Lorentz-covariant super-propagators (see the transition from the eq-n (122) to the eq-n (123)) to come back to Lorentz-covariant incomplete superpropagators from cutoff ones; (3) after these replacements we remark that the pair of considered vertices, connected by Lorentz-covariant incomplete superpropagator, represents the sum of Lorentz-covariant subdiagrams of the 2nd order, which can be replaced (as shown above, before the eq-n (120)) by the sum of corresponding LF subdiagrams plus the "vertex" generated by the counterterm (96), because, by the definition of the first class, the x^+ coordinates of the vertices in that pair do not coincide with the x^+ coordinates of other vertex points. Of course, the generalized tadpole diagrams, which can be attached to any of the vertices and are collected as factors before the considered sums of subdiagrams, should be taken into account separately.

Thus we get

$$S_n = \lim_{\alpha \to 0} \lim_{\varepsilon \to 0} \hat{S}_n, \qquad (126)$$

where the $\hat{S}_n$ is the sum of connected LF diagrams (as defined at the beginning of Sect. 4) generated by the Lagrangian (45) (they are, of course, not generalized tadpoles and do not contain generalized tadpole subdiagrams) with following modifications: firstly, to each superpropagator connecting the different type vertices of these diagrams the contribution, caused by the inclusion of the counterterm (96) to the action (see Sect. 5), is added, and, secondly, all generalized tadpole diagrams from Lorentz-covariant perturbation theory are added.

We remind also that the limit $\alpha \to 0$ should be taken only after summing all contributions to Green functions when this limit becomes finite.

Let us show that the sum of connected LF diagrams of the nth order, generated by the action (97), coincides with the quantity $\hat{S}_n$ at some choice of the B_n. Consider LF perturbation theory in the parameter $|B|$ for the action (97). We obtain the same set of diagrams, as with the Lagrangian (45), but (1) we have the factors iB and iB^* instead of the factors $i\frac{\gamma}{2}e^{i\theta}$ and $i\frac{\gamma}{2}e^{-i\theta}$ in the vertices, (2) the quantity (95) is added to every propagator (as a result of taking into account the term quadratic in B in the action). Owing to this addition some connected generalized tadpole diagrams, having the order,

higher, than the 1st, and equal to zero in usual LF perturbation theory, may become nonzero. One can show that only one generalized tadpole diagram of the 2nd order in $|B|$ is nonzero, and it is equal to

$$i|B|^2 w \qquad (127)$$

(that follows strightforwardly from the (95) and from taking into account the vertex factors iB and iB^*). These diagrams will be attached to vertices of both types (to those which have factors iB and iB^*), but no more than one diagram to each vertex. To get this result one has to proceed analogously to the proof of the fact that all usual connected LF generalized tadpole diagrams (higher than of 1st order) are equal to zero (see, for example, [21]), and take into account that the contributions resulting from simultaneous addition of the quantity (95) to several propagators, outgoing from one point, always equals to zero (see the eq-n (91)).

Thus starting from the action (97) we can get all cases, when generalized tadpole subdiagrams arise, in the following way: one has to attach to any vertex the generalized tadpole diagram of the 2nd order in $|B|$ that equals to the quantity (127) (this 2nd order includes the vertex factor of the mentioned vertex). If we consider the sum of all contributions, corresponding to a diagram with such attachments to some of its vertices and without this attachments, we see that both contributions can be described formally as only one, without the attachments, but with vertex factors iB and iB^* replaced by the

$$iB + i|B|^2 w, \quad and \quad iB^* + i|B|^2 w. \qquad (128)$$

Let us now choose the B so that

$$B + |B|^2 w = A, \qquad (129)$$

where the iA is the sum of connected generalized tadpole diagrams with all external lines attached to the vertices of the 1st type, generated by the Lagrangian (45) in Lorentz-covariant approach. Then the result of the extraction of the nth order in γ from the set of connected diagrams of lastly considered perturbation theory with the action (97) (the B is the series (98) in γ) coincides with $\hat{S}_n$, and, hence, in the limit $\varepsilon \to 0$, $\alpha \to 0$ is equal to the sum of Lorentz-covariant connected diagrams S_n of order n generated by the Lagrangian (45) (according to the eq-n (126)).

Thus we have shown that LF perturbation theory for connected Green functions, generated by the action (97), is equivalent in the $\varepsilon \to 0$, $\alpha \to 0$ limit

to Lorentz-covariant perturbation theory, corresponding to the Lagrangian (45). The coincidence of connected Green functions leads to the coincidence of Green functions without vacuum loops. The value of the quantity B, necessary for this coincidence, is given by the solution of the equation (129). This equation is fulfilled trivially in the 1st order in γ ($\gamma B_1 = \gamma A_1 = \frac{\gamma}{2}e^{i\theta}$), while in the 2nd order it coincides with the equation (119), obtained via straightforward analysis of the 2nd order diagrams. As it was shown above (see after the formulae (119)), the quantity B_2 is finite. The consideration of the eq-n (129) in the next orders shows that the B_k with $k > 2$ are divergent at removed regularization (namely, in the limit $\varepsilon\alpha \to 0$).

The eq-n (129) can be solved straightforwardly without using the perturbation theory. We get the following result:

$$B = -\frac{1}{2w} + \sqrt{\frac{1}{4w^2} + \frac{A'}{w} - A''^2} + iA'', \tag{130}$$

where $A = A'+iA''$, whereas the quantities A', A'' are real. The sign before the root is fixed by the requirement of the correspondence with the perturbation theory in γ. Let us remark that at large γ (i. e. at large fermion mass M) the argument of the root in the eq-n (130) may become negative, so that the eq-n (129) can not be solved. That's why our perturbative approach may be applicable only for sufficiently small M.

We know that the quantity iA (which is the sum of connected Lorentz-covariant generalized tadpole diagrams, generated by the Lagrangian (45), with the external lines attached to the vertex of the 1st type) diverges in the $\varepsilon\alpha \to 0$ limit only in the 2nd order in γ (see Sect. 3). According to eq-n (119) we have

$$A = \frac{\gamma^2}{4}w + const \tag{131}$$

(let us remind that $w \to \infty$ at $\varepsilon\alpha \to 0$). Here the *const* represent a series in γ with finite (in $\varepsilon \to 0$, $\alpha \to 0$ limit) terms starting from γA_1. Using the eq-n (131), one can take the limit $\varepsilon\alpha \to 0$ (and, therefore, the $w \to \infty$ limit) in the eq-n (130), whereas this can be done only beyond the perturbation theory. We get:

$$B = \sqrt{\frac{\gamma^2}{4} - A''^2} + iA'' = \frac{\gamma}{2}e^{i\hat{\theta}}, \qquad \sin\hat{\theta} = \frac{2A''}{\gamma}. \tag{132}$$

As it is seen from the eq-n (131), the quantity A'' is finite at $\varepsilon\alpha \to 0$. Remark that the formula (132) can not be used in the action (97), because the limit

$\alpha \to 0$ can not be taken due to the divergency of the term quadratic in the B. Therefore one should preserve the regularization in this action and use there, instead of the eq-n (132), the expression (130), where the B is the function of the initial parameters of the theory, γ and θ, and also of the regularization factor $\varepsilon\alpha$. The quantity B depends (through the quantity w) also on the function $v(z)$, which is not fixed completely, so that some arbitrariness remains. The LF Hamiltonian, corresponding to the action (97), has the form:

$$H = \int dx^- \left(\frac{1}{8\pi} m^2 : \varphi^2 : -B : e^{i\varphi} : -B^* : e^{-i\varphi} : \right) - 2\pi e^{-2C} \frac{|B|^2}{m^2} \times$$

$$\times \int dx^- dy^- \left(: e^{i\varphi(x^-)} e^{-i\varphi(y^-)} : -1 \right) \theta(|x^- - y^-| - \alpha) \frac{v(\varepsilon(x^- - y^-))}{|x^- - y^-|}, \qquad (133)$$

where the LF cutoffs $|p_-| \geq \varepsilon > 0$ are implied, and $\alpha > 0$. If the function $v(x)$ fulfills the introduced requirements and if the coefficient B, dependent on $v(x)$, is defined by the eq-n (130), then the LF Hamiltonian (133) generates perturbation theory which is equivalent in the limit $\varepsilon \to 0$, $\alpha \to 0$ to Lorentz-covariant perturbation theory corresponding to the Lagrangian (45).

7. The transformation of boson variables in the corrected LF Hamiltonian to canonical LF fermion variables

In this section we introduce LF fermionic fields, giving their construction in terms of boson field variables, i. e. we make the transformation inverse to the bosonization. In analogy with the procedure of Sect. 2 we consider the theory on finite interval $-L \leq x^- \leq L$ assuming periodic boundary conditions for the boson field $\varphi(x) = \sqrt{4\pi}\Phi(x)$. We replace the cutoff $|p_-| \geq \varepsilon > 0$ by simple exclusion of zero mode in the Fourier series

$$\varphi(x^-) = \sum_{n \neq 0} \varphi_n e^{i\frac{\pi}{L}nx^-}. \qquad (134)$$

The Hamiltonian (133) takes the form

$$H = \int_{-L}^{L} dx^- \left(\frac{1}{8\pi} m^2 : \varphi^2 : -B : e^{i\varphi} : -B^* : e^{-i\varphi} : \right) - 2\pi e^{-2C} \frac{|B|^2}{m^2} \times$$

$$\times \int_{-L}^{L} dx^- \int_{-L}^{L} dy^- \left(: e^{i\varphi(x^-)} e^{-i\varphi(y^-)} : -1 \right) \tilde{\theta}(x^- - y^-, \alpha) \frac{v\left(\frac{x^- - y^-}{L}\right)}{|x^- - y^-|}. \qquad (135)$$

Here the $\tilde{\theta}(z, \alpha)$ is periodic analog of the function $\theta(|z| - \alpha)$, i. e. $\tilde{\theta}(z, \alpha) = 0$, if $2Ln - \alpha < z < 2Ln + \alpha$ for some integer n, and otherwise $\tilde{\theta}(z, \alpha) = 1$. The function $v(z)$ also must be limited by the condition of the periodicity of the integrand (what means the translation invariance of the Hamiltonian in x^-) in the following way

$$v(z) = |z|\tilde{v}(z), \tag{136}$$

where the function $\tilde{v}(z)$ is periodic with the period equal to 2. Below we use the formulae similar to (35) to construct the LF version of fermion field in terms of boson field variables. With this aim we rewrite the normal ordered expression of the eq-n (135) in the form:

$$: e^{i\varphi(x^-)}e^{-i\varphi(y^-)} :=: e^{i\varphi(x^-)} :: e^{-i\varphi(y^-)} : e^{-\beta(x^- - y^-)}, \tag{137}$$

where

$$\beta(x^- - y^-) = \sum_{n=1}^{\infty} \frac{1}{n} e^{-i\frac{\pi}{L}n(x^- - y^-)}. \tag{138}$$

The construction, similar to the formulae (35), should represent only one fermionic field component on the LF (which remains independent after solving LF canonical constraints). Choosing the antiperiodic boundary conditions for this fermionic field we write the following expression satisfying canonical anticommutation relations on the LF as a consequence of ones for boson variables:

$$\hat{\psi}_+(x) = \frac{1}{\sqrt{2L}} e^{-i\hat{\omega}} e^{-i\frac{\pi}{L}x^- \hat{Q}} e^{i\frac{\pi}{2L}x^-} : e^{-i\varphi(x)} : . \tag{139}$$

Here the $\hat{\psi}_+$, $\hat{Q}$, $\hat{\omega}$ denote the LF analog of the quantities ψ_+, Q_+, ω_+, appearing in the bosonization procedure in Lorentz coordinates (described in Sect. 2), with the analogous commutation relations, but on the LF. We also fix the irrelevant now variable A_- to be equal to zero.

The described construction can be used to express the boson Hamiltonian in terms of fermion field on the LF. This expression takes a form similar to the naive canonical LF QED_2 Hamiltonian (when antiperiodic boundary conditions for fermion field and the condition $A_- = 0$ are chosen), if one defines the $\tilde{v}(z)$ as follows:

$$\tilde{v}(z) = \exp\left(\sum_{m=1}^{\infty} \frac{1}{m} e^{-i\pi mz}\right) \sum_{n=-\infty}^{\infty} \frac{1}{n + \frac{1}{2}} e^{i\pi nz}. \tag{140}$$

This function satisfies all necessary conditions. The condition, required at $z = 0$, can be checked using the following asymptotic forms:

$$\exp\left(\sum_{m=1}^{\infty} \frac{1}{m} e^{-i\pi m z}\right) = \frac{1}{1 - e^{-i\pi z}} = \frac{1}{i\pi z} + u_1(z), \tag{141}$$

$$\sum_{n=-\infty}^{\infty} \frac{1}{n + \frac{1}{2}} e^{i\pi n z} = 2 e^{-i\frac{\pi}{2} z} \left(\ln\left(1 - e^{-i\frac{\pi}{2} z}\right) - \ln\left(1 - e^{i\frac{\pi}{2} z}\right)\right) +$$

$$+ \ln\left(1 - e^{i\pi z}\right) - e^{-i\pi z} \ln\left(1 - e^{-i\pi z}\right) = i\pi \text{sign}(z) + u_2(z), \tag{142}$$

where $u_1(z)$, $u_2(z)$ are continuous functions at $z = 0$ and $u_2(0) = 0$. Hence the function $v(z) = |z| \tilde{v}(z)$ is continuous and equal to unity at $z = 0$. As remarked at the end of the preceding section this is sufficient for having perturbation theory equivalent to the Lorentz-covariant one in the limit $\varepsilon \to 0$, $\alpha \to 0$.

The Hamiltonian (135) can be rewritten as follows:

$$H = \int_{-L}^{L} dx^- \left(\frac{1}{8\pi} m^2 : \varphi^2 : - B : e^{i\varphi} : - B^* : e^{-i\varphi} : \right) - 2\pi e^{-2C} \frac{|B|^2}{m^2 L} \times$$

$$\times \int_{-L}^{L} dx^- \int_{-L}^{L} dy^- : e^{i\varphi(x^-)} : : e^{-i\varphi(y^-)} : \sum_{n=-\infty}^{\infty} \frac{\tilde{\theta}(x^- - y^-, \alpha)}{n + \frac{1}{2}} e^{i\frac{\pi}{L} n(x^- - y^-)}. \tag{143}$$

Returning to fermion variables according to the formulae (139) one gets:

$$H = \int_{-L}^{L} dx^- \left(\frac{e^2}{2} \left(\partial_-^{-1}[\hat{\psi}_+^+ \hat{\psi}_+]\right)^2 - \sqrt{2L} \left(B^* e^{-i\frac{\pi}{2L} x^-} e^{i\frac{\pi}{L} x^- \hat{Q}} e^{i\hat{\omega}} \hat{\psi}_+ + h.c.\right)\right) -$$

$$- 4\pi e^{-2C} \frac{|B|^2}{m^2} \int_{-L}^{L} dx^- \int_{-L}^{L} dy^- \left(\sum_{n=-\infty}^{\infty} \frac{1}{n + \frac{1}{2}} e^{i\frac{\pi}{L} n(x^- - y^-)}\right) \tilde{\theta}(x^- - y^-, \alpha) \times$$

$$\times \hat{\psi}_+^+(x^-) e^{-i\hat{\omega}} e^{-i\frac{\pi}{L} x^- \hat{Q}} e^{i\frac{\pi}{2L} x^-} e^{-i\frac{\pi}{2L} y^-} e^{i\frac{\pi}{L} y^- \hat{Q}} e^{i\hat{\omega}} \hat{\psi}_+(y^-), \tag{144}$$

where the $[f(x)]$ denotes the quantity $f(x)$ without its zero mode in x^- like in the eq-n (7). Using commutation relations between $\hat{\psi}_+$, $\hat{Q}$, $\hat{\omega}$, which are analogous to ones for ψ_+, Q_+, ω_+ in Sect. 2, and restricting the Hamiltonian to physical subspace with $\hat{Q} = 0$, we get

$$H = \int_{-L}^{L} dx^- \left(\frac{e^2}{2} \left(\partial_-^{-1}[\hat{\psi}_+^+ \hat{\psi}_+]\right)^2 - \sqrt{2L} \left(B^* e^{-i\frac{\pi}{2L} x^-} e^{i\hat{\omega}} \hat{\psi}_+ + h.c.\right)\right) -$$

$$- 4\pi e^{-2C} \frac{|B|^2}{m^2} \int_{-L}^{L} dx^- \int_{-L}^{L} dy^- \left(\sum_{n=-\infty}^{\infty} \frac{1}{n + \frac{1}{2}} e^{i\frac{\pi}{L}(n + \frac{1}{2})(x^- - y^-)}\right) \tilde{\theta}(x^- - y^-, \alpha) \times$$

$$\times \hat{\psi}_+^+(x^-) \hat{\psi}_+(y^-), \tag{145}$$

38

It is easy to show that the multiplication of the quantity (142) in the formulae (145) by the function $\tilde{\theta}(x^- - y^-, \alpha)$ can (at small α) modify noticeably the Fourier modes of this quantity only at $n \sim \frac{L}{\alpha}$. However, since the calculations with LF Hamiltonian are always made at some finite value of the p_-, this modification can not change the results. This means that one can take the limit $\alpha \to 0$ in the Hamiltonian (145) (let us remark that after this one can not develop the perturbation theory in γ (i. e. in fermion mass), because this means actually the returning to bosons where one has divergency at $\alpha = 0$). We conclude that one can use, in particular, the finite expression (132) for the quantity B instead of the expression (130). Then one can rewrite the Hamiltonian (145) as follows (using eq-n (47)):

$$H = \int\limits_{-L}^{L} dx^- \Big(\frac{e^2}{2} \Big(\partial_-^{-1} [\hat{\psi}_+^+ \hat{\psi}_+] \Big)^2 -$$

$$-\frac{eMe^C\sqrt{2L}}{4\pi^{3/2}} \Big(e^{-i\hat{\theta} - i\frac{\pi}{2L}x^-} e^{i\hat{\omega}} \hat{\psi}_+ + h.c. \Big) - \frac{iM^2}{2} \hat{\psi}_+^+ \partial_-^{-1} \hat{\psi}_+ \Big). \qquad (146)$$

The quantity $\hat{\theta}$, determined by the eq-n (132), can be interpreted as some condensate angle. Indeed, one can show (as in the paper [33]) that

$$A = \frac{1}{2} \Big(\gamma \frac{\partial}{\partial \gamma} + \frac{1}{i} \frac{\partial}{\partial \theta} \Big) \tilde{G}_0, \qquad (147)$$

where the $\tilde{G}_0$ is the connected vacuum Green function density:

$$\tilde{G}_0 = \frac{1}{V} \ln\langle 0|T \exp\Big(i \int d^2 x L_I\Big) |0\rangle. \qquad (148)$$

Here the V is the volume of the space-time, and the L_I is given by the formula (47). Therefore one can write in Heisenberg representation:

$$A = \frac{\gamma}{2} \langle \Omega| : e^{i(\varphi + \theta)} : |\Omega\rangle, \qquad (149)$$

$$\sin\hat{\theta} = \langle \Omega| : \sin(\varphi + \theta) : |\Omega\rangle = -\frac{2\pi^{3/2}}{e\, e^C} \langle \Omega| : \bar{\Psi}\gamma^5\Psi : |\Omega\rangle, \qquad (150)$$

where the $|\Omega\rangle$ is the physical vacuum, and the normal ordering is taken with respect to the bare vacuum of Lorentz-covariant perturbation theory. The quantity $\hat{\theta}$ is a finite function of the parameters M/e and θ of initial theory (see the text after the formulae (132)). According to the first form of the formulae

(150) the quantity $\hat{\theta}$ coincides with the θ in lowest order in γ (i. e. at $M = 0$). If the fermion mass M (actually M/e) increases, the right hand side of the eq-n (150) may become greater than unity. Then this eq-n cannot be resolved with respect to $\hat{\theta}$, and our method of the construction of LF Hamiltonian, using the perturbation theory in M, becomes not working. This was already remarked after the eq-n (130). Only numerical nonperturbative calculations can tell us something more concrete about the appearance of such difficulty.

Let us remark that the last term in the formulae (146) coincides with the term, which usually appears in direct quantization on the LF (after solving canonical constraints for fermion fields).

If one writes the expression for $\hat{\psi}(x)_+$ in terms of canonical fermion creation and annihilation operators at $x^+ = 0$:

$$\hat{\psi}_+(x) = \frac{1}{\sqrt{2L}} \left(\sum_{n \geq 1} b_n e^{-i\frac{\pi}{L}(n-\frac{1}{2})x^-} + \sum_{n \geq 0} d_n^+ e^{i\frac{\pi}{L}(n+\frac{1}{2})x^-} \right), \tag{151}$$

so that

$$\{b_n, b_{n'}^+\} = \{d_n, d_{n'}^+\} = \delta_{nn'}, \quad b_n|0\rangle = d_n|0\rangle = 0, \tag{152}$$

and the momentum P_- and the charge $\hat{Q}$ take the form

$$P_- = \sum_{n \geq 1} b_n^+ b_n \frac{\pi}{L}\left(n - \frac{1}{2}\right) + \sum_{n \geq 0} d_n^+ d_n \frac{\pi}{L}\left(n + \frac{1}{2}\right), \tag{153}$$

$$\hat{Q} = \sum_{n \geq 1} b_n^+ b_n - \sum_{n \geq 0} d_n^+ d_n, \tag{154}$$

one can express the 2nd term in the eq-n (146) using only fermionic "zero" modes (after the integration over x^-):

$$P_+ = H = \int\limits_{-L}^{L} dx^- \left(\frac{e^2}{2} \left(\partial_-^{-1}[\hat{\psi}_+^+ \hat{\psi}_+] \right)^2 - \right.$$
$$\left. - \frac{M}{2} \left(R\, e^{i\hat{\omega}} d_0^+ + h.c. \right) - \frac{iM^2}{2} \hat{\psi}_+^+ \partial_-^{-1} \hat{\psi}_+ \right), \tag{155}$$

where

$$R = \frac{e\, e^C}{2\pi^{3/2}} e^{-i\hat{\theta}}. \tag{156}$$

The complex parameter R can be related with fermion condensates as follows:

$$|R| = \left|\langle\Omega| : \bar{\Psi}\left(1 + i\gamma^5\right)\Psi : |\Omega\rangle\right|_{M=0},$$
$$\text{Im } R = \langle\Omega| : \bar{\Psi}\gamma^5\Psi : |\Omega\rangle. \tag{157}$$

In contrast to our previous paper [33] only finite condensates enter our LF Hamiltonian explicitly. This also agrees with general idea of the paper [42] that only finite nonperturbative (with respect to usual coupling) parts of the condensates should play role of physically meaningful parameters. One can use the M and the $\hat{\theta}$ as independent parameters in nonperturbative calculations with our LF Hamiltonian and fit these parameters to reproduce known results.

Let us remark that the phase operator $e^{i\hat{\omega}}$, present in the LF Hamiltonian (155), is defined on the LF analogously to formulas (32-34), with the LF analog of "filled" states (17). This means that we have

$$[\hat{\omega}, \hat{Q}] = i, \qquad e^{i\hat{\omega}}\hat{Q}e^{-i\hat{\omega}} = \hat{Q} - 1, \tag{158}$$

and therefore, using the formulae (139), we can write

$$e^{i\hat{\omega}}\hat{\psi}_+(x)e^{-i\hat{\omega}} = e^{i\frac{\pi}{L}x^-}\hat{\psi}_+(x), \tag{159}$$

so that

$$e^{i\hat{\omega}}b_n e^{-i\hat{\omega}} = b_{n+1}, \quad e^{i\hat{\omega}}d_n^+ e^{-i\hat{\omega}} = d_{n-1}^+, \qquad n \geq 1,$$
$$e^{i\hat{\omega}}d_0^+ e^{-i\hat{\omega}} = b_1. \tag{160}$$

Besides we have

$$e^{i\hat{\omega}}|0\rangle = b_1^+|0\rangle, \qquad e^{-i\hat{\omega}}|0\rangle = d_0^+|0\rangle, \tag{161}$$

because the vacuum $|0\rangle$ has the following "Dirac see" form on the LF:

$$|0\rangle = \left(\prod_{n=0}^{\infty} d_n\right)|0_D\rangle. \tag{162}$$

These equalities help to calculate matrix elements of the Hamiltonian (155) on physical subspace, defined by the condition

$$\hat{Q}|phys\rangle = 0. \tag{163}$$

As simplest example one can consider $q\bar{q}$ approximation for boson bound state wave functions and compare the results of the calculation of the spectrum with known ones.

It is more interesting, however, to find a way of generalization of our 2-dimensional analysis to 4-dimensional QCD. Such possibility can be connected with the assumption that at small M nonperturbative contributions related with condensates enter LF Hamiltonian by means of only zero modes of quark and gluon fields with the coefficients depending on condensate parameters in analogy with considered here LF QED_2. This assumption was already used in our earlier paper [15]. Similar idea was discussed also in [35, 36].

8. Conclusion

In conclusion let us formulate the results. We considered the problem of finding LF Hamiltonian which gives the theory equivalent to conventional one (in Lorentz-covariant formulation). For the models with gauge symmetry this problem usually is very difficult one. In the present paper we solved it successfully for QED in (1+1)-dimensions. The following methods were used: (1) we transformed the QED_2 to its bosonized version, which is known theory of selfinteracting scalar field (with fermion mass M playing the role of coupling constant), (2) in this bosonized theory we applied the method proposed earlier to find the difference between LF and Lorentz-covariant perturbation theories (in small parameter M), what gives us the counterterm in LF Hamiltonian that can compensate this difference, (3) then we made "inverse" transformation from boson variables to initial fermionic ones directly on the LF (in fact we have made the transformation to only that component of fermion field which remains independent after solving canonical LF constraints for fermion fields). In the obtained LF Hamiltonian (145) we used the formulation with discretized LF momentum (more convenient for nonperturbative calculations), choosing antiperiodic boundary conditions for fermion fields (while starting with periodic ones for the boson field) on the LF interval $|x^-| \leq L$. The UV-type regularization is present in this Hamiltonian, and the coefficients before the counterterms are perturbative series in fermion mass with terms diverging in the limit of removing the regularization. However, after the summation of these series, the mentioned coefficients turn out to be finite in the limit of removing the regularization. This allowed us to formulate the LF Hamiltonian in the final form (155), where the limiting values for these coefficients were used. In this form the Hamiltonian can not be longer used for the construction of perturbation theory in M. But it can be applied for nonperturbative calculations.

This final LF Hamiltonian has, beside of usual terms, obtained via naive canonical LF quantization of QED_2 in LF gauge, a counterterm, proportional to linear combination of fermionic zero modes $(\psi_+^+)_0$ and $(\psi_+)_0$ (multiplied by some operator phase factors neutralizing their charge and fermionic number), with coefficients before them $(-MR/2$ and $-MR^*/2$, correspondingly) proportional to fermion mass M and depending on fermion condensates:

$$|R| = \left|\langle\Omega| : \bar{\Psi}\left(1 + i\gamma^5\right)\Psi : |\Omega\rangle\right|\big|_{M=0} = \frac{e\,e^C}{2\pi^{3/2}},$$
$$\operatorname{Im}R = \langle\Omega| : \bar{\Psi}\gamma^5\Psi : |\Omega\rangle = -|R|\sin\hat{\theta}. \tag{164}$$

Such final LF Hamiltonian generates a theory equivalent to Lorentz-covariant QED_2 (in the $L \to \infty$ limit), if the fermion mass M is small enough to guarantee the consistency of the eq-ns (164).

One can use the M and the $\hat{\theta}$ as input parameters in nonperturbative (numerical) calculations with the obtained LF Hamiltonian and fit the $\hat{\theta}$ to known spectra.

The form of the obtained counterterm is rather simple and allows to hope that for 4-dimensional QCD LF Hamiltonian the effects, related with condensates, can be taken into account with only zero modes of fields like in the QED_2 (at least at small M and semi-phenomenologically).

Appendix 1

Perturbative form of Green functions in terms of the superpropagators.

The N-point Green function $G_N(y_1, \ldots, y_N)$ has the following form in the interaction picture:

$$G_N(y_1, \ldots, y_N) = \langle 0|T\left(\varphi(y_1)\ldots\varphi(y_N)e^{i\int d^2x L_I}\right)|0\rangle, \tag{A1.1}$$

where T is "time" ordering symbol and

$$L_I = \frac{\gamma}{2}e^{i\theta} : e^{i\varphi} : +\frac{\gamma}{2}e^{-i\theta} : e^{-i\varphi} : . \tag{A1.2}$$

We expand the exponent in perturbation theory series:

$$G_N(y_1,\ldots,y_N) = \sum_{l,m=0}^{\infty} \left(\frac{i\gamma}{2}\right)^{l+m} \frac{e^{i(l-m)\theta}}{l!m!} \int \left(\prod_{i=1}^{l+m} d^2x_i\right) \times$$

$$\times \langle 0|T\left(\prod_{n=1}^{N}\varphi(y_n)\right)\left(\prod_{i=1}^{l} : e^{i\varphi(x_i)} :\right)\left(\prod_{j=1}^{m} : e^{-i\varphi(x_{l+j})} :\right)|0\rangle. \qquad (A1.3)$$

At any fixed order of times in the time-ordered product we can calculate corresponding vacuum matrix element by transposing the exponents with only creation operators to the left and those with annihilation operators to the right. The result can be expressed in terms of propagators $\Delta(x)$ as follows [32]:

$$G_N(y_1,\ldots,y_N) = \sum_{l,m=0}^{\infty} \left(\frac{i\gamma}{2}\right)^{l+m} \frac{e^{i(l-m)\theta}}{l!m!} \int \left(\prod_{i=1}^{l+m} d^2x_i\right) \times$$

$$\times \left(\prod_{i,j=1;i<j}^{l} e^{-\Delta(x_i-x_j)}\right)\left(\prod_{i,j=1;i<j}^{m} e^{-\Delta(x_{l+i}-x_{l+j})}\right)\left(\prod_{i=1}^{l}\prod_{j=1}^{m} e^{\Delta(x_i-x_{l+j})}\right) \times$$

$$\times \langle 0|T\prod_{n=1}^{N}\left(\varphi(y_n) + i\sum_{j=1}^{l+m}\Delta(y_n-x_j)\text{sign}(l-j+\frac{1}{2})\right)|0\rangle. \qquad (A1.4)$$

This formulae shows that perturbation theory series is expressed actually in terms of superpropagators $e^{\pm\Delta(x)}$, which connect "internal" points having coordinates x_i, and Lorentz-covariant propagators, which connect these points with the "external" points, denoted by the y_i.

Appendix 2

Let us describe briefly the method of the paper [21] allowing to compare LF and Lorentz-covariant Feynman integrals in the theory with polynomial interaction. Consider, for example, an arbitrary 1-loop Feynman diagram with external entering momenta p_μ^i, $i = 1, 2, \ldots$. The loop momentum k_- is bounded by cutoff conditions $|k_- - \sum p_-^i| \geq \varepsilon$, steming from the restriction on the propagator momenta. On the other side, analogous covariant diagram contains the integration over all k. Therefore the difference between these diagrams can be found as the sum of integrals over the bands $|k_- - \sum p_-^i| < \varepsilon$.

Let us estimate one of these "ε"-band integrals. We shift the variable k_- in this integral so that $|k_-| < \varepsilon$. Then we change the scale:

$$k_- \longrightarrow \varepsilon k_-, \qquad k_+ \longrightarrow \frac{1}{\varepsilon}k_+. \qquad (A2.1)$$

that makes the integration interval independent of ε, while keeps unchanged Lorentz-invariant products like k_+k_- or dk_+dk_-. Propagators, corresponding to internal lines whose momenta are outside of the ε-band (owing to external momentum p_- going through the line) change as follows:

$$\frac{i}{\pi}\frac{1}{2(k_+ + p_+)(k_- + p_-) - m^2 + i0} \longrightarrow$$

$$\longrightarrow \frac{i}{\pi}\frac{1}{2(\frac{1}{\varepsilon}k_+ + p_+)(\varepsilon k_- + p_-) - m^2 + i0} \underset{\varepsilon \to 0}{\approx} \frac{i}{\pi}\frac{\varepsilon}{2k_+ p_-}. \qquad (A2.2)$$

In the paper [21] we used denotations for the lines with momenta outside and inside of the ε-band. The first one was called Π-line and the last one ε-line. It follows from the eq-n(A2.2) that every Π-line gives a factor of order $O(\varepsilon)$ while every ε-line gives a factor of order $O(1)$. Therefore the integral over the band is zero in $\varepsilon \to 0$ limit if at least one of Π-lines is present in the corresponding diagram.

Similar analysis can be made for an arbitrary many-loop Feynman diagram. The difference between LF and covariant calculation of this diagram can be estimated again by considering all possible configurations of Π- and ε- lines in the diagram [21]. It was shown in the paper [21] (for a wide class of field theories) that each of these configurations can be estimated as having the order $O(\varepsilon^\sigma)(1 + O(\log \varepsilon))$ with respect to ε, where

$$\sigma = min(\tau, \omega_- - \omega_+ - \mu + \eta). \qquad (A2.3)$$

Here the minimum is to be taken w.r.t. all subdiagrams of the diagram at some configuration of Π- and ε- lines in it; $\omega_\pm$ are indices of UV-divergency in $k_\pm$ of a given subdiagram; μ is the index of total UV-divergency in k_- of all Π-lines in the subdiagram; τ is the total power of the ε that arises, after the change $k_- \to \varepsilon k_-$ of loop variables k_-, from numerators of all propagators of the diagram and from all volume elements in the integrals over k_-; η is the part of the τ related with only those numerators and volume elements (used in the definition of τ) that are not present in the considered subdiagram. Let us apply this general result to our scalar field theory. All propagators have simple structure. Only possible contribution to the τ comes from the volume elements dk_-. Because we are interested only in the difference of LF and covariant diagrams, any configuration should contain at least one integration over k_- in the ε-band. Therefore, $\tau > 0$ (and $\eta \geq 0$). Due to Lorentz-invariant form of diagrams in k_+, k_- we have $\omega_+ - \omega_- = 0$. It

follows from the expression (A2.2) for a Π-line propagator that the μ can be counted as the number of Π-lines in the subdiagram taken with the minus sign. Therefore, one has $-\mu + \eta > 0$ (and, hence, $\sigma > 0$) if at least one of Π-lines is present. Thus, only configurations without Π-lines, i. e. at $\mu = \eta = \sigma = 0$, can contribute to the difference between LF and covariant diagrams. The absence of the Π-lines means that all external lines of the diagram are attached to only one vertex. We call such diagrams generalized tadpole. The logarithmic corrections, that are mentioned before the eq-n (A2.3) and can arise in general, are present, in fact, only if they depend on external momenta. This is related with the fact that owing to the Lorentz invariance the argument of the logarithm can contain the ε only if it contains also a p_--component of external momenta. However the generalized tadpole diagrams do not depend on external momenta, therefore logarithmic corrections are absent.

We see that the difference between the Lorentz-covariant and LF Feynman integrals in considered theory with polynomial interaction is caused only by generalized tadpoles.

References

[1] P.A.M. Dirac. Rev.Mod.Phys., 21 (1949) 392.

[2] K.G. Wilson, T.S. Walhout, A. Harindranath, W.-M. Zhang, R.J. Perry, S.D.Glazek. Phys.Rev. D49 (1994) 6720.

[3] S.J. Brodsky, H.-C. Pauli, S.S. Pinsky. Phys.Rep., 301 C (1998), or hep-ph/9705477, and references therein.

[4] V.A. Franke, Yu.V. Novozhilov, S.A. Paston, E.V. Prokhvatilov, in book: Quantum Theory in honour of Vladimir A. Fock, Part 1; Unesco, St. Petersburg University, Euro-Asian Physical Society, (1998) 38-97. Hep-th/9901029.

[5] A.M. Annenkova, E.V. Prokhvatilov, V.A. Franke. Solving Schroedinger equation for Sine-Gordon model in Light-Front coordinates. Vestnik Leningradskogo Universiteta, N4 (1985) 80.

[6] A.B. Bylev, E.V. Prokhvatilov, V.A. Franke. Vestnik Leningradskogo Universiteta, ser.4, N2 (1986) 8.

[7] T. Eller, H.-C. Pauli, S.J. Brodsky. Phys.Rev., D35 (1987) 1493.

[8] J.R. Klauder, H. Leutwyler, L. Streit. Nuovo Cim., 66A (1970) 536.

[9] T. Maskawa, K. Yamawaki. Progr. Theor. Phys., 56 (1976) 270.

[10] V.A. Franke, Yu.V. Novozhilov, E.V. Prokhvatilov. Lett. Math. Phys., 5 (1981) 239.

[11] V.A. Franke, Yu.V. Novozhilov, E.V. Prokhvatilov. Lett. Math. Phys., 5 (1981) 437.

[12] V.A. Franke, Yu.V. Novozhilov, E.V. Prokhvatilov. Vestnik Leningrad-skogo Universiteta, N22 (1981) 13.

[13] V.A. Franke, Yu.V. Novozhilov, E.V. Prokhvatilov. Dynamical systems and microphysics. Geometry and mechanics. Proc. of the 2nd Intern. Seminar, Udine, Italy, September 1981. Academic Press, 1982, p. 389.

[14] A.B. Bylev, E.V. Prokhvatilov, V.A. Franke. Vestnik Leningradskogo Universiteta, ser.4,issue 2, N11 (1989) 66.

[15] E.V. Prokhvatilov, V.A. Franke. Sov. J. Nucl. Phys., 49 (1989) 688.

[16] E.V. Prokhvatilov, H.W.L. Naus, H.-J. Pirner. Phys.Rev., D51 (1995) 2933. Hep-th/9406275.

[17] S.J. Chang, R.G. Root, T.M. Yan. Phys.Rev., D7 (1973) 1133; S.J.Chang, T.M. Yan. Phys. Rev., D7 (1973) 1147.

[18] M. Burkardt, A. Langnau. Phys. Rev., D44 (1991) 1187, 3857.

[19] V.A. Franke, E.V. Prokhvatilov. Theory of Hadrons and Light-Front QCD. Proc. of the 4th Intern. Workshop on Light-Front Quantization and Non-perturbative Dynamics. Polana Zgorzolisko, Poland, August 1994. World Scientific Publishing, (1995) 272.

[20] E.V. Prokhvatilov, V.A. Franke. Phys. Atom. Nucl., 59 (1996) 2030.

[21] S.A. Paston, V.A. Franke. Theor. Math. Phys. 112, N3 (1997) 1117-1130. Hep-th/9901110.

[22] S.A. Paston, V.A. Franke, E.V. Prokhvatilov. Theor. Math. Phys. 120, N3 (1999) 1164-1181. Hep-th/0002062.

[23] H.J. Lowenstein, J.A. Swieca. Ann. Phys., 68 (1971) 172.

[24] S. Coleman. Phys.Rev., D11 (1975) 2088.

[25] S. Coleman, R.Jackiw, L.Susskind. Ann.Phys., 93 (1975) 267.

[26] S. Coleman. Ann.Phys.m 101 (1976) 239.

[27] L. Affleck. Nucl.Phys., B265 (1986) 409.

[28] N.S. Manton. Ann.Phys., 159 (1985) 220.

[29] Y. Nakawaki. Prog. Theor.Phys., 70 (1983) 1105.

[30] E.V. Prokhvatilov. Theor. Math. Phys., 88 (1991) 685.

[31] R. Jackiw. Rev. Mod. Phys., 52 (1980) 661.

[32] C. Adam. Annals Phys. 259 (1997) 1-63. Hep-th/9704064.

[33] S.A. Paston, E.V. Prokhvatilov, V.A. Franke. Hep-th/9910114.

[34] M. Burkardt. Phys. Rev., D47 (1993) 4628.

[35] G. McCartor. Phys. Rev., D60 (1999) 105004.

[36] G. McCartor. Hep-th/0008040.

[37] D.A. Uhlenbrock. Comm. Math. Phys., 4 (1967) 64.

[38] S. Weinberg. Phys. Rev. 118 (1960) 838.

[39] W.-M. Zhang, A. Harindranath. Phis. Rev. D48 (1993) 4868, 4881, 4903.

[40] N. E. Ligterink, B. L. G. Bakker. Phis. Rev. D52 (1995) 5954.

[41] Yu.V. Novozhilov, E.V. Prokhvatilov, V.A. Franke. In book: The problems of theoretical physics, part III. Leningrad, LGU, 1988. P. 5-21.

[42] V. A. Novikov, M. A. Shifman, A. I. Vainstein. Phys. Rep. C 116 (1984) 103.